CAPACITY-DEMAND-DIAGRAM METHODS FOR ESTIMATING DEFORMATION OF INELASTIC SYSTEMS

Anil K. Chopra
and
Rakesh K. Goel

In collaboration with Degenkolb Engineers
ATC-40 Nonlinear Static Procedure

1. Develop the pushover curve
ATC-40 Nonlinear Static Procedure

2. Convert pushover curve to capacity diagram

\[ A = \frac{V_b}{M_1^*} \]

\[ D = \frac{U_N}{\Gamma_1 \phi_{N1}} \]
3. Plot elastic design spectrum in A-D format

\[
D = \frac{T_n^2}{4 \pi^2} A
\]

Demand Diagram
ATC-40 Nonlinear Static Procedure

4. Plot the demand diagram and capacity diagram together
   Intersection point gives displacement demand
   Avoids nonlinear RHA; instead analyze equivalent linear systems
ATC-40 Nonlinear Static Procedure

5. Convert displacement demand to roof displacement and component deformation.
6. Compare to limiting values for specified performance goals.
Objectives

- Examine the procedure to determine the deformation of SDF systems
  - Does the iterative procedure converge?
  - How accurate is the estimated deformation?
  - Damping modification in ATC-40?
- Develop improved procedure
  - Based on inelastic design spectrum
  - Gives deformation consistent with design spectrum
Evaluation Method

- Define equivalent linear system
  - Period based on secant stiffness
  - Equivalent damping ratio
- Estimate deformation by ATC-40 procedures
  - Ground motion
  - Design spectrum
- Compare estimated deformation with “exact” value from nonlinear RHA or reference value from design spectrum
Equivalent Period

- For bilinear systems
  \[ T_{eq} = T_n \sqrt{\frac{\mu}{1 + \alpha \mu - \alpha}} \]

- For elasto-plastic systems
  \[ T_{eq} = T_n \sqrt{\mu} \]
Equivalent Damping

- For bilinear systems
  \[ \zeta_{eq} = \frac{2 (\mu - 1)(1 - \alpha)}{\pi \mu (1 + \alpha \mu - \alpha)} \]

- For elasto-plastic systems
  \[ \zeta_{eq} = \frac{2 (\mu - 1)}{\pi \mu} \]
Variation of Period and Damping of Equivalent Linear System with Ductility
ATC-40 Procedure

- Damping modification
  \[ \zeta_{eq} = \zeta + \kappa \zeta_{eq} \]
- \( \kappa \) depends on
  - Structural behavior
    - Type A
    - Type B
    - Type C
  - Judgement

![Graph showing \( \kappa \) values for different types of structural behavior.](image)
1. Plot the capacity diagram and 5%-damped elastic demand diagram.
2. Estimate deformation demand $D_i$ and determine $A_i$ from the capacity spectrum. Initially, assume $D_i = D(T_n, \zeta = 5\%)$.
3. Compute $\mu = D_i \div D_y$.
4. Compute $\zeta_{eq} = \zeta + \kappa \zeta_{eq}$.
5. Plot the elastic demand diagram for $\zeta_{eq}$, intersection with capacity diagram gives displacement $D_j$.
6. Check for convergence.
   
   If $(D_j - D_i) \div D_j \leq$ Tolerance then $D = D_j$.
   
   Otherwise, set $D_i = D_j$ and repeat steps 3 to 6.
Examples: Specified Ground Motion
ATC-40 Procedure-A Analysis of System 5

- Given: $T_n = 1 \text{ s}$, $\zeta = 0.05$, $f_y/w = A_y/g = 0.10$.
- Input: 1940 El Centro Ground Motion
- Find: $D_{\text{approx}}$
  1. Capacity and 5%-damped elastic demand diagram
  2. Start at $D = 11.3 \text{ cm}$
ATC-40 Procedure-A Analysis of System 5

3. $\mu = \frac{11.3}{2.56} = 4.40$

4. $\zeta_{eq} = \frac{2}{\pi} \times (\frac{1}{\mu} - \frac{1}{\mu}) = 0.49 = 0.45$
   
   $k = 0.77$

   $\zeta_{eq} = 0.05 + 0.77 \times 0.45 = 0.397$

5. Capacity diagram intersects 39.7%-damped elastic demand diagram at 3.73 cm
ATC-40 Procedure-A Analysis of System 5

6. Error = \(100 \times \frac{3.73 - 11.3}{3.72} = -202.6\% > 5\%\) tolerance

Repeat steps 3 to 6 till convergence is achieved

\(D_{\text{approx}} = 4.46 \text{ cm}\)

\(D_{\text{exact}} = 10.2 \text{ cm}\)

Error = -56.1\%
ATC-40 Procedure-A Analysis of System 5

5% Demand Diagram

29.7% Demand Diagram (Last Iteration)

39.7% Demand Diagram (1st Iteration)

Capacity Diagram
The ATC-40 Procedure A converges to a deformation much smaller than the “exact” value.

Convergence is deceptive because it may leave the erroneous impression that calculated deformation is accurate.
ATC-40 Procedure-A Analysis of System 6

- Given: $T_n = 1$ s, $\zeta = 0.05$, $f_y \div w = A_y \div g = 0.17$.
- Input: 1940 El Centro Ground Motion
- Find: $D_{\text{approx}}$
- Procedure-A fails to converge
ATC-40 Procedure-A Analysis of System 6

- ATC-40 Procedure A fails to converge.
- Intersection point oscillates between 11.7 and 3.52 cm.
- Procedure diverges even when restarted with deformation close to “exact” value.
Examples: Design Spectrum
Newmark-Hall Elastic Design Spectrum

- Acceleration Sensitive
- Velocity Sensitive
- Displacement Sensitive

- $T_a = 1/33$ sec
- $T_b = 0.125$ sec
- $T_c$
- $T_d$
- $T_e = 10$ sec
- $T_f = 33$ sec
ATC-40 Procedure-A Analysis of System 5

- Given: \( T_n = 1 \text{ s}, \ \zeta = 0.05, \ f_y \div w = A_y \div g = 0.45. \)
- Input: Newmark-Hall (1982) design spectrum
- Find: \( D_{approx} \)
  1. Capacity and 5%-damped elastic demand diagram
  2. Start at \( D = 44.6 \text{ cm} \)
ATC-40 Procedure-A Analysis of System 5

3. $\mu = 44.6 \div 11.6 = 4.0$

4. $\zeta_{eq} = \left(\frac{2}{\pi}\right) \times \left(\frac{\mu - 1}{\mu}\right) = 0.48 = 0.45$
   
   $k = 0.77$

   $\zeta_{eq} = 0.05 + 0.77 \times 0.45 = 0.397$

5. Capacity diagram intersects 39.7%-damped elastic demand diagram at 28.2 cm
6. Error $= 100 \times \frac{(28.2 - 44.6)}{44.6} = -58.4\% > 5\%$ tolerance

Repeat steps 3 to 6 till convergence is achieved

$D_{\text{approx}} = 30.4 \text{ cm}$

$D_{\text{exact}} = 44.6 \text{ cm}$

Error $= -31.8\%$
ATC-40 Procedure-A Analysis of System 5
ATC-40 Procedure-A Analysis of System 5

- ATC-40 Procedure A gives deformation smaller than inelastic design spectrum.
- Convergence leaves an erroneous impression that calculated deformation is accurate.
ATC-40 Procedure-A Analysis of System 6

- Given: $T_n = 1 \text{ s}$, $\zeta = 0.05$, $f_y / w = A_y / g = 0.90$.
- Input: Newmark-Hall (1982) design spectrum
- Find: $D_{\text{approx}}$
- Procedure-A fails to converge
ATC-40 Procedure-A Analysis of System 6

- ATC-40 Procedure A fails to converge.
- Intersection point oscillates between 13.72 and 89.28 cm.
- Procedure diverges even when restarted with deformation close to value determined from inelastic design spectrum
ATC-40 Procedure B

1. Plot the capacity diagram.
2. Estimate deformation demand $D_i$. Initially, assume $D_i = D(T_n, \zeta = 5\%)$.
3. Compute $\mu = \frac{D_i}{D_y}$.
4. Compute $T_{eq}$ and $\zeta_{eq}$.
5. Compute $D(T_{eq}, \zeta_{eq})$ and $A(T_{eq}, \zeta_{eq})$ of an equivalent SDF system with $T_{eq}$ and $\zeta_{eq}$.
6. Plot $D(T_{eq}, \zeta_{eq})$ and $A(T_{eq}, \zeta_{eq})$.
7. Check if the curve generated by connecting points plotted in step 6 intersects the capacity diagram.
   If yes, intersection point gives $D$.
   Otherwise, repeat steps 3 to 7.
Examples: Ground Motion
ATC-40 Procedure-B Analysis of Systems 4 to 6
Application of ATC-40 Procedure B to Systems 5 and 6

- **System 5**
  
  \[ D_{\text{exact}} = 10.16 \text{ cm} \]
  \[ D_{\text{arrox}} = 4.45 \text{ cm} \]
  
  Error = -56.2%

- **System 6**
  
  \[ D_{\text{exact}} = 8.53 \text{ cm} \]
  \[ D_{\text{arrox}} = 5.32 \text{ cm} \]
  
  Error = -37.7%
Examples: Design Spectrum
ATC Procedure-B Analysis of Systems 4 to 6
ATC-40 Procedure-B Analysis of Systems 5 and 6

- **System 5**
  \[ D_{\text{exact}} = 44.6 \text{ cm} \]
  \[ D_{\text{arrox}} = 30.4 \text{ cm} \]
  Error = -31.7%

- **System 6**
  \[ D_{\text{exact}} = 44.6 \text{ cm} \]
  \[ D_{\text{arrox}} = 29.8 \text{ cm} \]
  Error = -33.1%
Evaluation of ATC-40 Procedure: Ground Motion
Comparison of deformations: $\mu = 2$

- ATC-40 method is inaccurate
  - Underestimates deformation significantly over a wide period range
  - $D \equiv$ Half of “Exact”
- Damping modification factor, $\kappa$, is not attractive
  - Results improve marginally
  - $\kappa$ based on judgement
Comparison of deformations: $\mu = 2$

![Graph showing comparison of deformations](graph.png)

- **Exact**
- **ATC–40, $\kappa=1$**
- **ATC–40**

The graph compares the deformations for different models and parameters, with a focus on $\mu = 2$. The x-axis represents time in seconds ($T_n$), and the y-axis represents deformation in centimeters ($D_{cm}$).
Errors in deformations: $\mu = 2$
Comparison of deformations: $\mu = 6$

- ATC-40 method is inaccurate
  - Underestimates deformation significantly over a wide period range
- Damping modification factor, $\kappa$, is not attractive
  - Results improve marginally
  - $\kappa$ based on judgement
Errors for Six Ground Motions

El Centro, Imperial Valley Earthquake (1940)

Corralitos, Loma Prieta Earthquake (1989)

Sylmar County Hospital, Northridge Earthquake (1994)

Pacoima Dam, San Fernando Earthquake (1971)

Lucern Valley, Landers Earthquake (1992)

SCT, Mexico City Earthquake (1985)
Evaluation of ATC-40 Procedure: Design Spectrum
Deformation Spectra for Inelastic Systems

- Newmark-Hall (NH)
- Krawinkler-Nassar (KN)
- Vidic, Fajfar, and Fischinger (VFF)
- NH and KN give similar results except for $T_n < 0.3$ sec
Comparison of Deformations (NH)

- Large errors in ATC-40
- Discrepancy depends on ductility and period region
  - Underestimates deformation in acc. and disp. regions; discrepancy increases with increasing $\mu$
  - Underestimates deformation in velo. region for $\mu = 2$ and 4, but overestimates for $\mu = 8$.
- ATC-40 method is deficient compared to **elastic** spectrum in velo. and disp. regions
Comparison of Deformations (NH)

![Graph showing comparisons of deformations with Design Spectrum (NH) and ATC-40.](attachment:image.png)

- Acceleration Sensitive
- Velocity Sensitive
- Displacement Sensitive

Design Spectrum (NH)

ATC-40

$\mu = 4$
Errors in Deformations (NH)
Results for NH, KN and VFF Design Spectra
Improved Procedures

• Existing procedures use equivalent linear systems
  ➡ Convergence of iterative procedure?
  ➡ Excessive damping?
  ➡ Large errors

• Develop procedures based on inelastic design spectrum
  ➡ Eliminate errors (or discrepancies)
  ➡ Retain graphical appeal
Definitions

- $f_o = \text{Strength demand for elastic behavior}$
- $f_y = \text{Yield strength}$
- Normalized yield strength
  \[ \overline{f}_y = \frac{f_y}{f_o} \]
- Yield strength reduction factor
  \[ R_y = \frac{f_o}{f_y} \]
Inelastic Design Spectrum

- Yield strength required for structure to remain elastic
  \[ f_o = \frac{A}{g} w \]
- Yield strength of inelastic structure
  \[ f_y = \frac{A_y}{g} w \]
- Yield strength reduction factor: \( R_y(\mu, T_n) \)
  \[ A_y = \frac{A}{R_y} \]
- Plot of \( A_y \)'s \( T_n \)
- \( R_y-\mu-T_n \) relations
  - Newmark-Hall (1982)
  - Krawinkler-Nassar (1992)
  - Vidic-Fajfar-Fischinger (1994)
  - Others
Inelastic Design Spectra
Inelastic Design Spectra Using Three Different $R_y - \mu - T_n$ Equations
Inelastic Demand Diagram

- Inelastic design spectrum plotted in A-D format
- Deformation from spectrum: \( D = m D_y = \mu \left( \frac{T_n}{2\pi} \right)^2 A_y \)
- Plot \( A_y \) v's \( D \) for constant \( \mu \)
Improved Procedure-A Analysis of System 5

- Plot capacity and demand diagrams in $A-D$ format
- Yielding branch of capacity diagram intersects the demand diagram for several $\mu$
- **At relevant intersection point, $\mu$ from the two diagrams should match**
- Interpolate between two $\mu$ values or plot demand diagrams at finer $\mu$ values if necessary
Improved Procedure-A Analysis of System 5
Improved Procedure-A Analysis of Systems
4 to 6
Improved Procedure-A Analysis of System 2

- Plot capacity and demand diagrams in A-D format
- Yielding branch of capacity diagram intersects the demand diagram for several $\mu$
- At relevant intersection point, $\mu$ from the two diagrams should match
- Interpolate between two $\mu$ values or plot demand diagrams at finer $\mu$ values if necessary
Improved Procedure-A Analysis of System 2
Improved Procedure-A Analysis of Systems 1 to 3
Improved Procedure-A Applied to Other Inelastic Design Spectra
Improved Procedure-A Applied to Other Inelastic Design Spectra
Improved Procedure-A

- Gives deformations consistent with the selected design spectrum
- Retains graphical feature of ATC-40 Procedure-A
  - Desired deformation is at intersection of capacity and demand diagrams
- Demand diagram used is different
  - Constant-$\mu$ demand diagram in improved procedure
  - Elastic demand diagram in ATC-40
Improved Procedure-B

- Plot capacity diagram
- Plot $D - Ay$ pairs to generate curve A-B
  - Assume expected $\mu$
  - Determine $Ay$ from inelastic design spectrum
  - Calculate $D$
- Find intersection of A-B and the capacity diagram
Improved Procedure-B

- Gives deformations consistent with the selected design spectrum
- Retains graphical feature of ATC-40 Procedure-B
  - Desired deformation is at intersection of curve A-B and the capacity diagram
- Different systems are analyzed to obtain a point on curve A-B
  - Inelastic system in improved procedure
  - Equivalent elastic system in ATC-40
Numerical Version of Improved Procedure

• Compute \( R_y = f_0 / f_y = A / A_y \)

• Determine \( \mu \) from available \( R_y - \mu - T_n \) relations
  - Newmark-Hall
  - Krawinkler-Nassar
  - Vidic-Fajfar-Fischinger
  - Others

• Estimate deformation demand: \( D = \mu D_y \)
Three Different $R_y - \mu - T_n$ Relations

- $R_y$ versus $T_n$ for selected $\mu$
- $\mu$ versus $T_n$ for selected $R_y$
Numerical Version of Improved Procedure Analysis of System 3

Given: $T_n = 0.5\text{s}$, $\zeta = 5\%$, $A_y = 1.56\ \text{g}$

$\Rightarrow A = 2.71\ \text{g}$ for system to remain elastic

$\Rightarrow D_y = 9.7\ \text{cm}$

Newmark-Hall Inelastic Design Spectrum

Find: $D$

1. $R_y = 2.71 \div 1.56 = 1.73$.
2. $\mu = \frac{(1 + 1.73^2)}{2} = 2.0$
   
   Obtained from Newmark-Hall $R_y - \mu - T_n$ relations

3. $D = 2.0 \times 9.7 = 19.4\ \text{cm}$
Conclusions

- ATC-40 method is inaccurate
  - Underestimates deformation significantly over a wide range of $T_n$ and $\mu$ values
  - $D \equiv \text{Half of “Exact”}$
- ATC-40 method is deficient compared to elastic spectrum in velocity and displacement regions
Conclusions

- Improved capacity-demand-diagram methods, based on well-known constant ductility design spectrum, have been developed
  - Improved Procedure-A
  - Improved Procedure-B
  - Numerical Version of Improved Procedure

- Improved procedures give deformation consistent with the selected design spectrum
Conclusions

- Improved Procedure-A retains graphical feature of ATC-40 Procedure-A
  - Desired deformation is at intersection of capacity and demand diagrams

- Demand diagram used is different
  - Constant-$\mu$ demand diagram in improved procedure
  - Elastic demand diagram in ATC-40
Conclusions

- Improved Procedure-B retains graphical feature of ATC-40 Procedure-B
  - Desired deformation is at intersection of curve A-B and the capacity diagram
- Different systems are analyzed to obtain a point on curve A-B
  - Inelastic system in improved procedure
  - Equivalent elastic system in ATC-40
Conclusions

- Numerical Version of Improved Procedure is convenient if graphical feature is not needed
  - Based on $R_y - \mu - T_n$ relations
  - Gives essentially the same values of deformation as graphical implementation