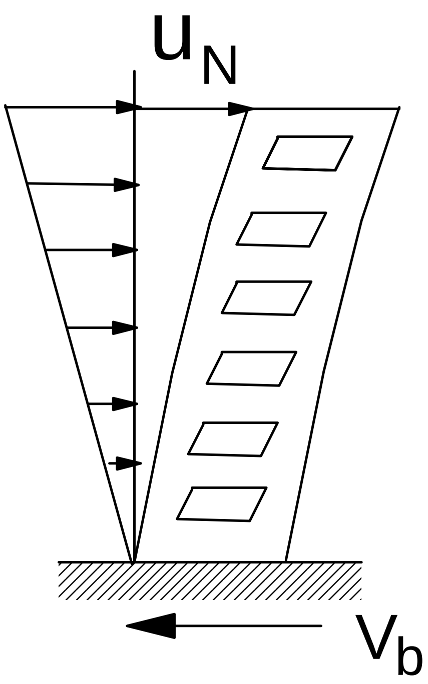
CAPACITY-DEMAND-DIAGRAM METHODS FOR ESTIMATING DEFORMATION OF INELASTIC SYSTEMS

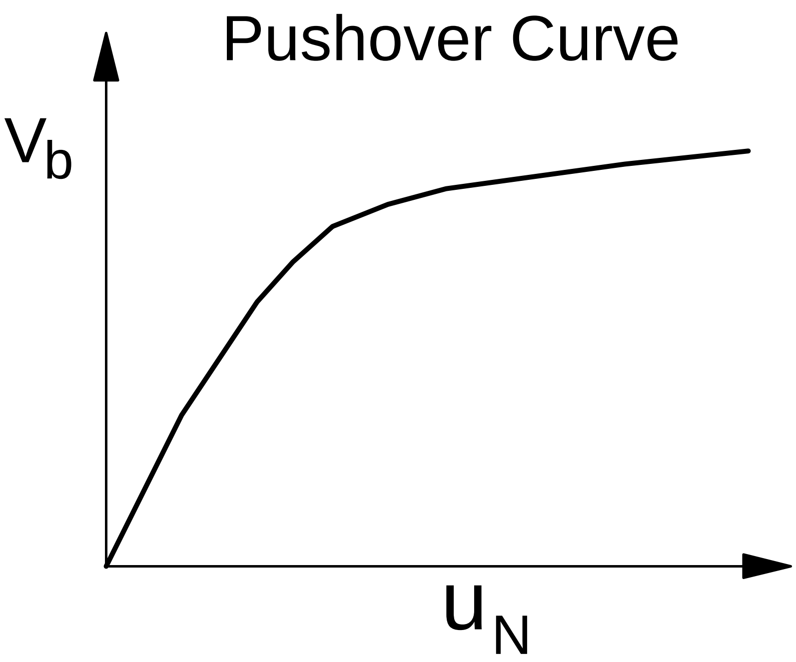
Anil K. Chopra
and
Rakesh K. Goel

In collaboration with Degenkolb Engineers

ATC-40 Nonlinear Static Procedure

1. Develop the pushover curve

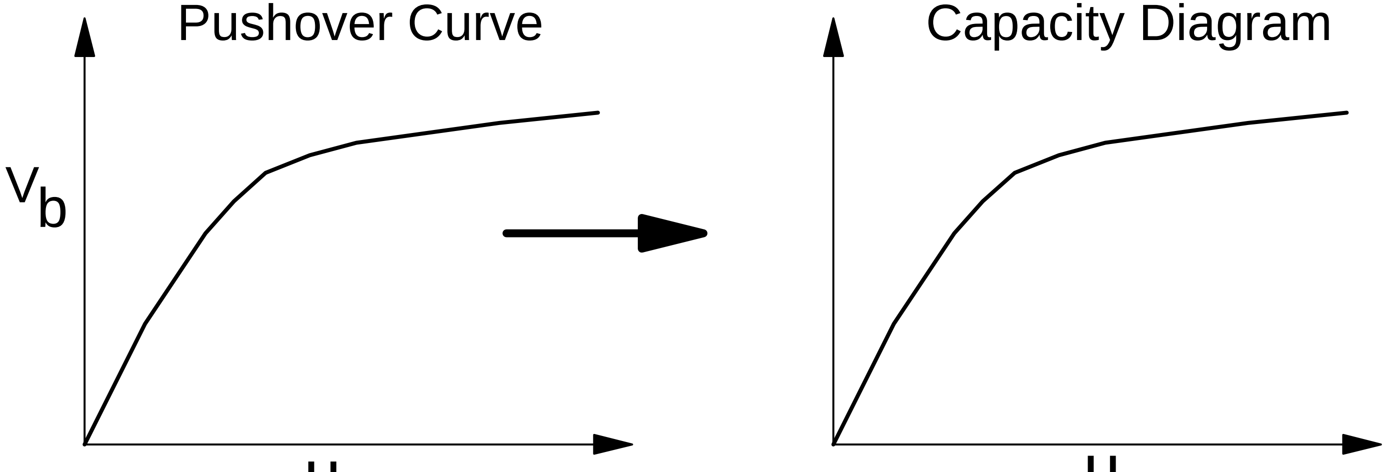




ATC-40 Nonlinear Static Procedure

2. Convert pushover curve to capacity diagram

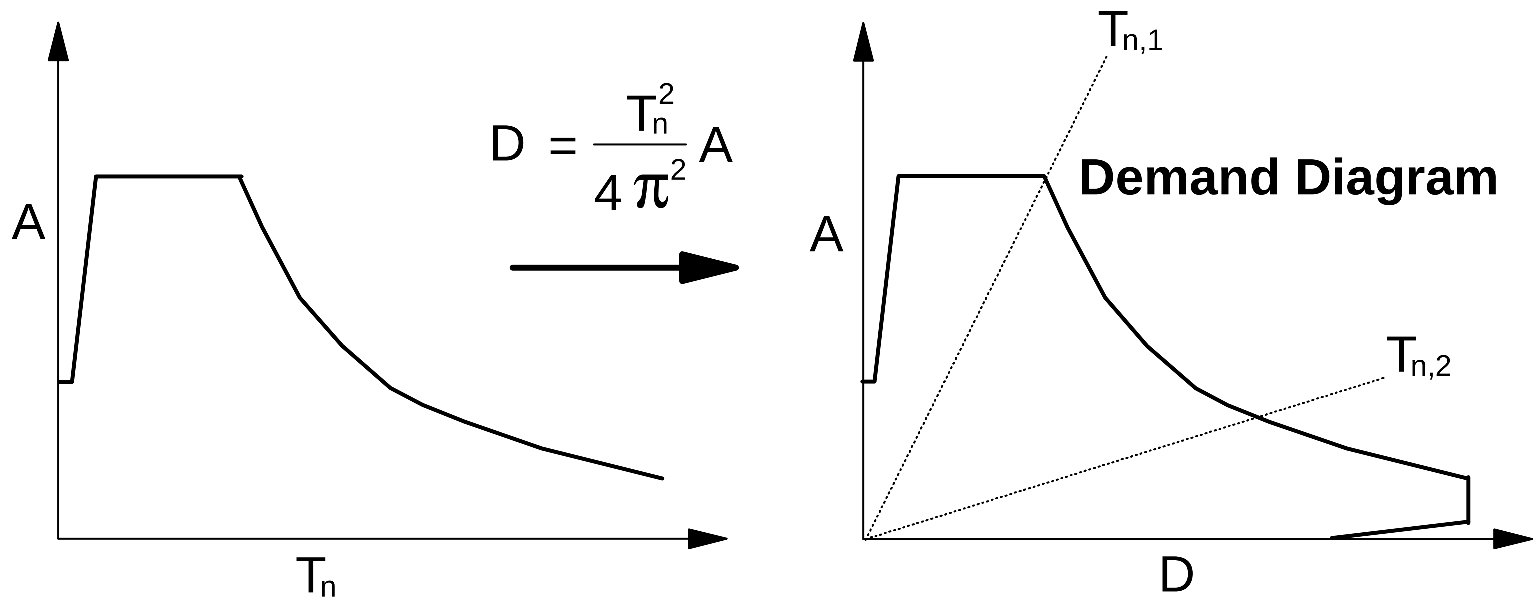
$$A = \frac{V_b}{M_1^*}$$



$$D = \frac{u_N}{\Gamma_1 \phi_{N1}}$$

ATC-40 Nonlinear Static Procedure

3. Plot elastic design spectrum in A-D format

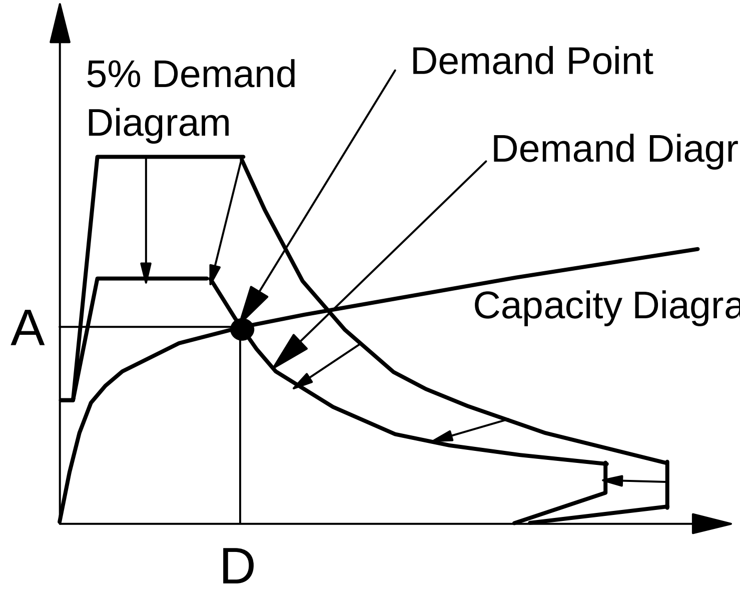


ATC-40 Nonlinear Static Procedure

4. Plot the demand diagram and capacity diagram together

   Intersection point gives displacement demand

   Avoids nonlinear RHA; instead analyze equivalent linear systems



ATC-40 Nonlinear Static Procedure

5. Convert displacement demand to roof displacement and component deformation.
6. Compare to limiting values for specified performance goals.

Objectives

- Examine the procedure to determine the deformation of SDF systems
  - ➔ Does the iterative procedure converge?
  - ➔ How accurate is the estimated deformation?
  - ➔ Damping modification in ATC-40?
- Develop improved procedure
  - ➔ Based on inelastic design spectrum
  - ➔ Gives deformation consistent with design spectrum

Evaluation Method

- Define equivalent linear system
  - ➔ Period based on secant stiffness
  - ➔ Equivalent damping ratio
- Estimate deformation by ATC-40 procedures
  - ➔ Ground motion
  - ➔ Design spectrum
- Compare estimated deformation with “exact” value from nonlinear RHA or reference value from design spectrum

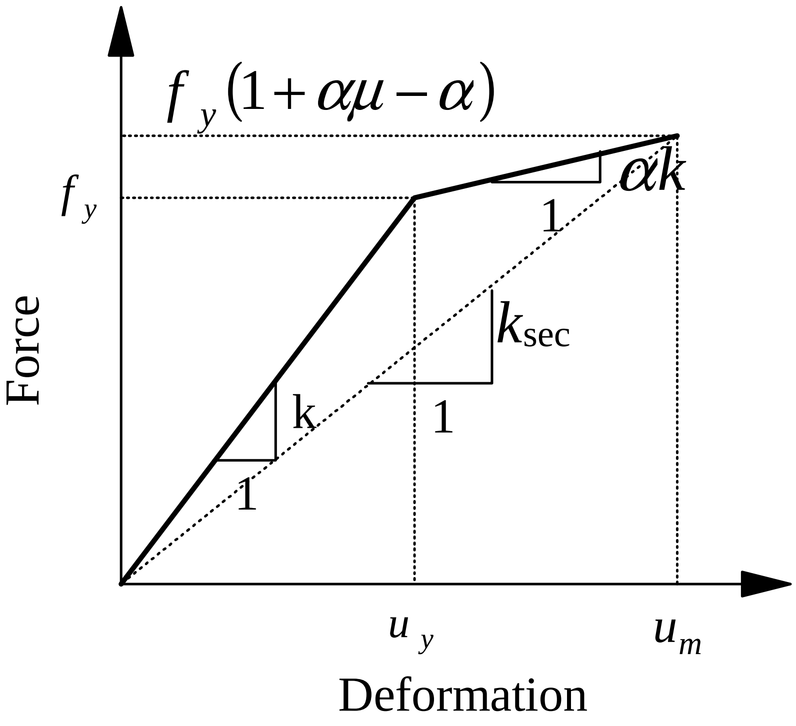
Equivalent Period

- For bilinear systems

$$T_{eq} = T_n \sqrt{\frac{\mu}{1 + \alpha\mu - \alpha}}$$

- For elasto-plastic systems

$$T_{eq} = T_n \sqrt{\mu}$$



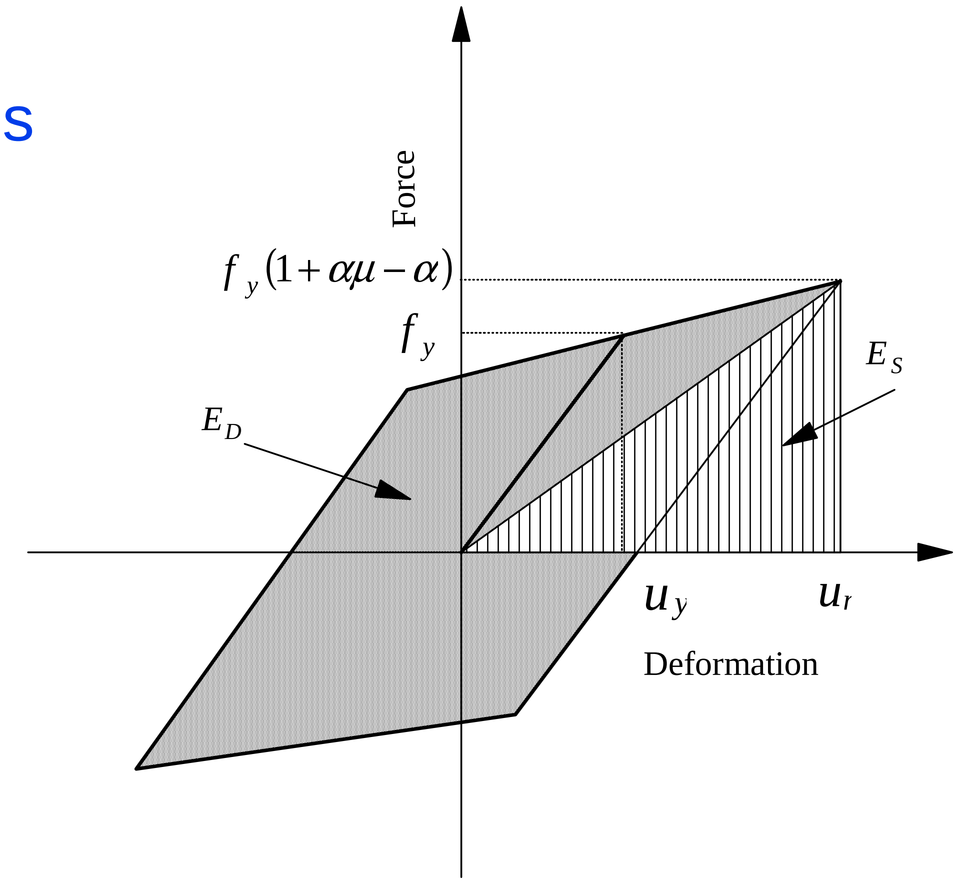
Equivalent Damping

- For bilinear systems

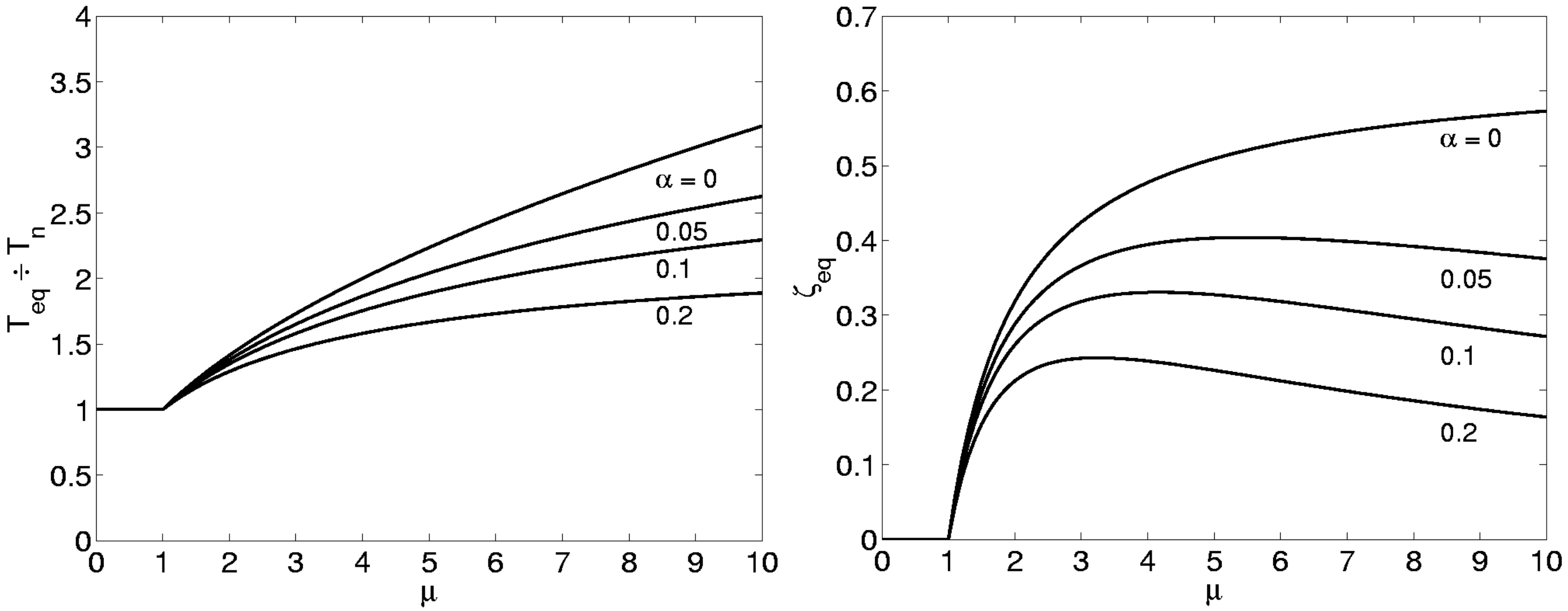
$$\zeta_{eq} = \frac{2}{\pi} \frac{(\mu - 1)(1 - \alpha)}{\mu(1 + \alpha\mu - \alpha)}$$

- For elasto-plastic systems

$$\zeta_{eq} = \frac{2}{\pi} \frac{(\mu - 1)}{\mu}$$



Variation of Period and Damping of Equivalent Linear System with Ductility

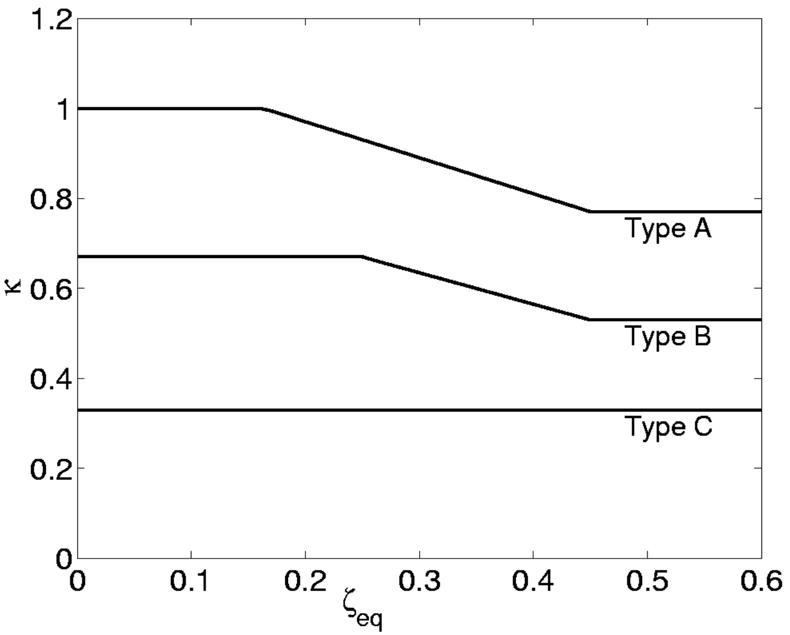


ATC-40 Procedure

- Damping modification

$$\hat{\zeta}_{eq} = \zeta + \kappa \zeta_{eq}$$

- κ depends on
  - ➔ Structural behavior
    - Type A
    - Type B
    - Type C
  - ➔ Judgement



ATC-40 Procedure A

1. Plot the capacity diagram and 5%-damped elastic demand diagram.
2. Estimate deformation demand D_i and determine A_i from the capacity spectrum. Initially, assume $D_i = D(T_n, \zeta = 5\%)$.
3. Compute $\mu = D_i \div D_y$.
4. Compute $\zeta_{eq} = \zeta + \kappa\, \zeta_{eq}$.
5. Plot the elastic demand diagram for ζ_{eq}, intersection with capacity diagram gives displacement D_j.
6. Check for convergence.

   If $(D_j - D_i) \div D_j \leq$ Tolerance then $D = D_j$.

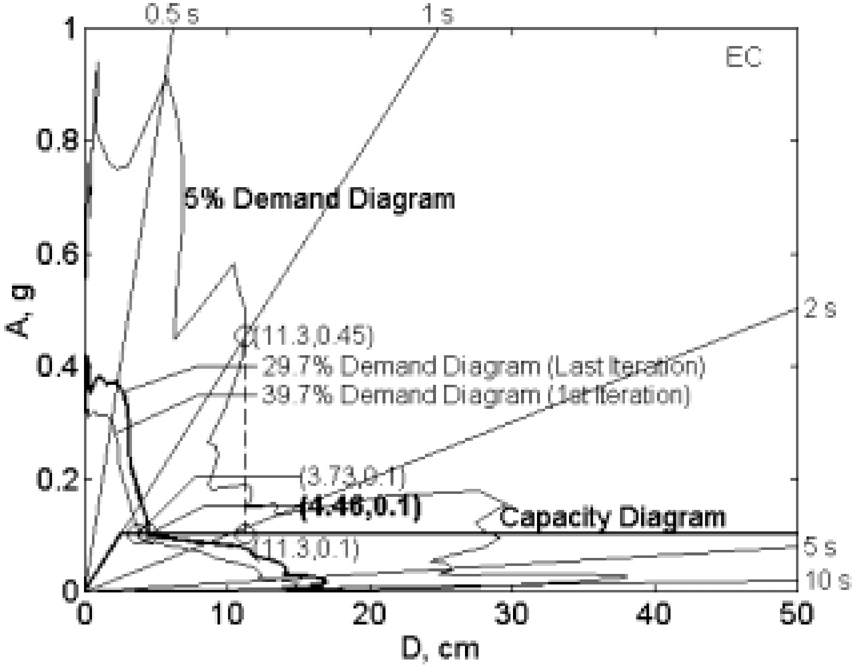
   Otherwise, set $D_i = D_j$ and repeat steps 3 to 6.

Examples: Specified Ground Motion

ATC-40 Procedure-A Analysis of System 5

- Given: T_n = 1 s, ζ = 0.05, $f_y \div w = A_y \div g$ = 0.10.
- Input: 1940 El Centro Ground Motion
- Find: D_{approx}

1. Capacity and 5%-damped elastic demand diagram
2. Start at D =11.3 cm



ATC-40 Procedure-A Analysis of System 5

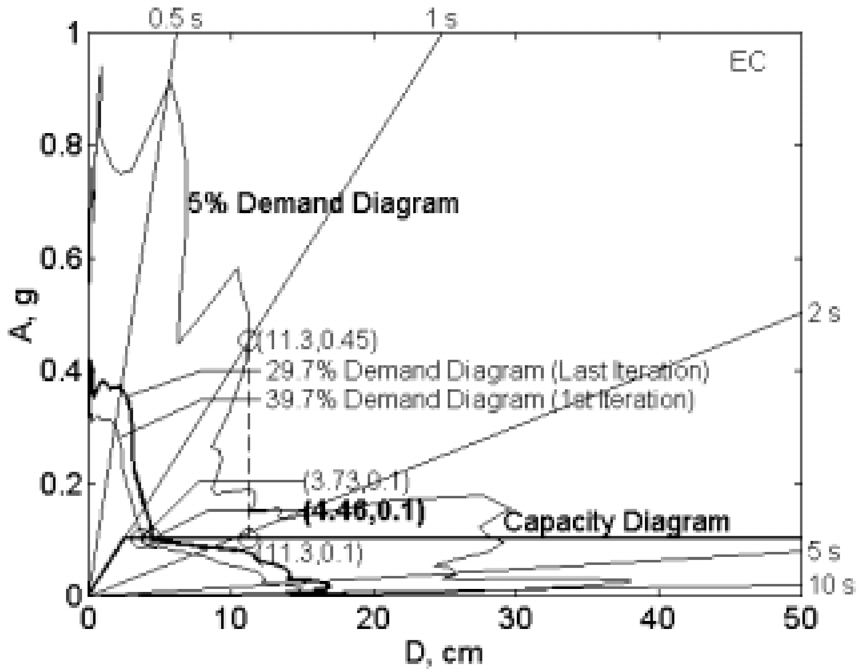
3. $\mu = 11.3 \div 2.56 = 4.40$

4. $\zeta_{eq} = (2/\pi) \times (\mu - 1/\mu) =$ ~~0.49~~ $= 0.45$

   $k = 0.77$

   $\zeta_{eq} = 0.05 + 0.77 \times 0.45 = 0.397$

5. Capacity diagram intersects 39.7%-damped elastic demand diagram at 3.73 cm



ATC-40 Procedure-A Analysis of System 5

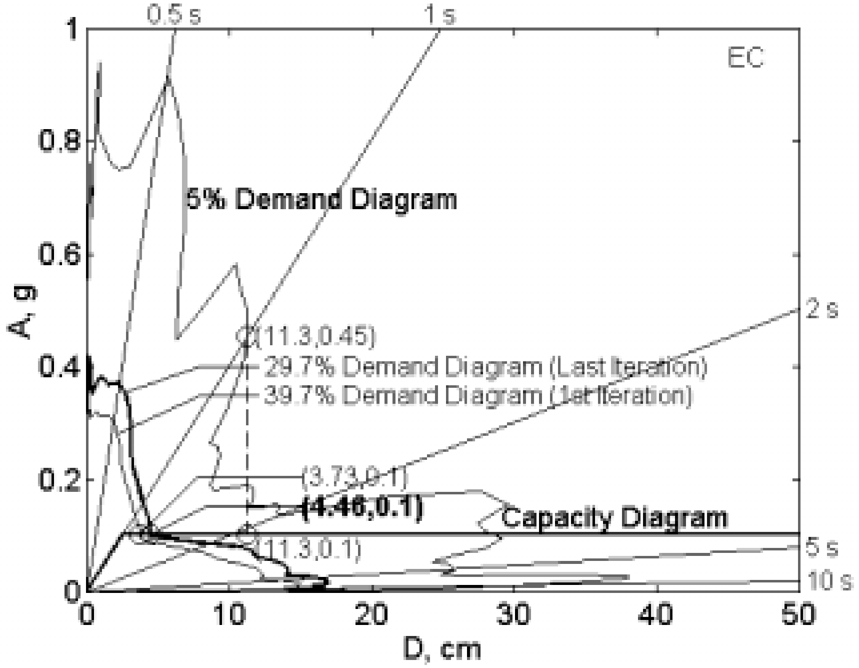
6. Error = 100×(3.73-11.3)÷3.72 = -202.6% > 5% tolerance

   Repeat steps 3 to 6 till convergence is achieved

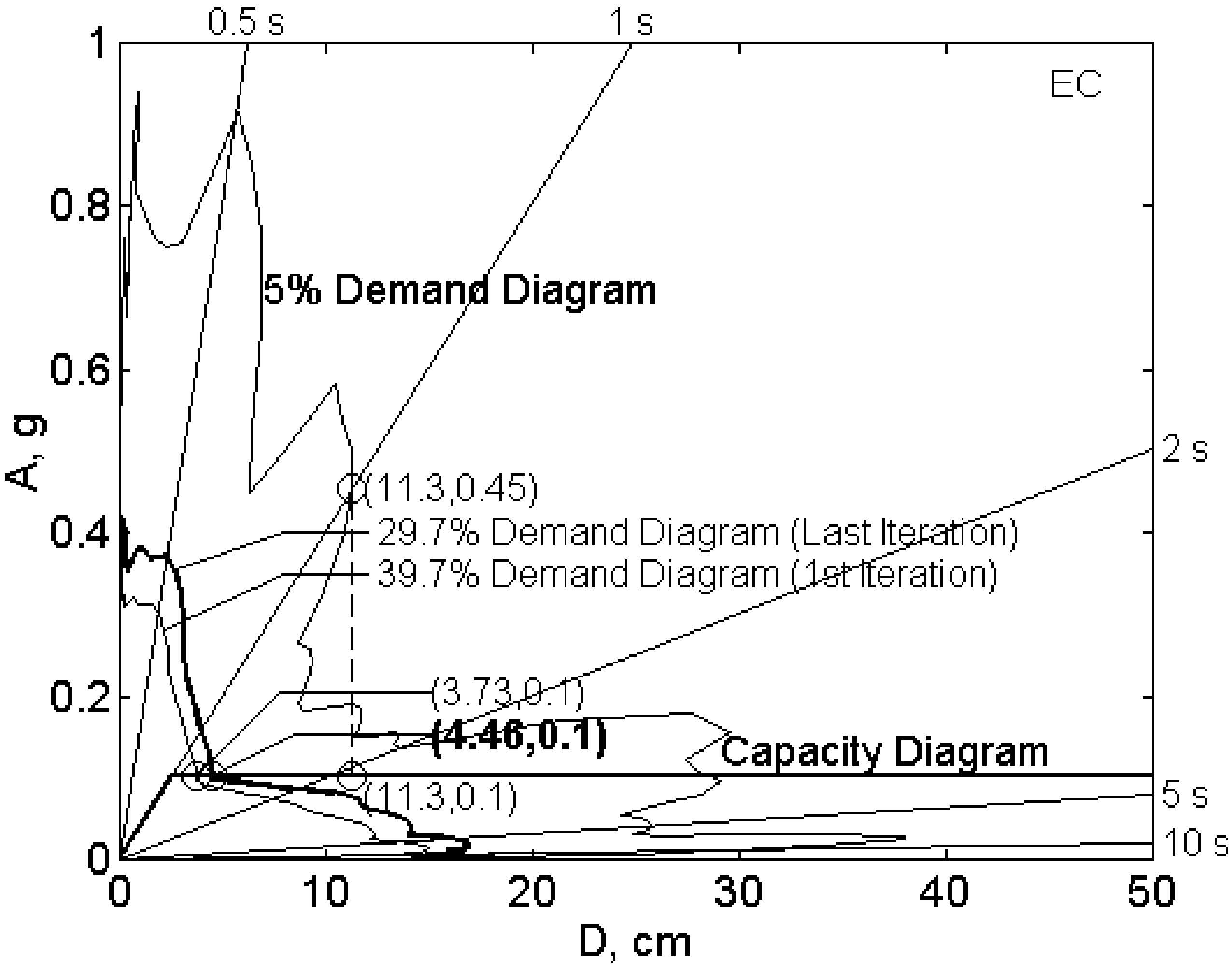
   D_{approx} = 4.46 cm

   D_{exact} = 10.2 cm

   Error = -56.1%

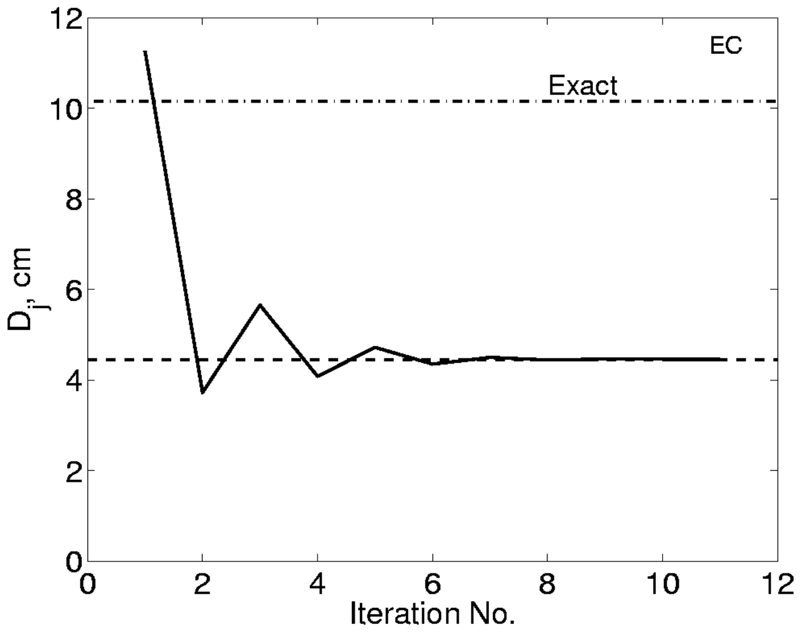


ATC-40 Procedure-A Analysis of System 5



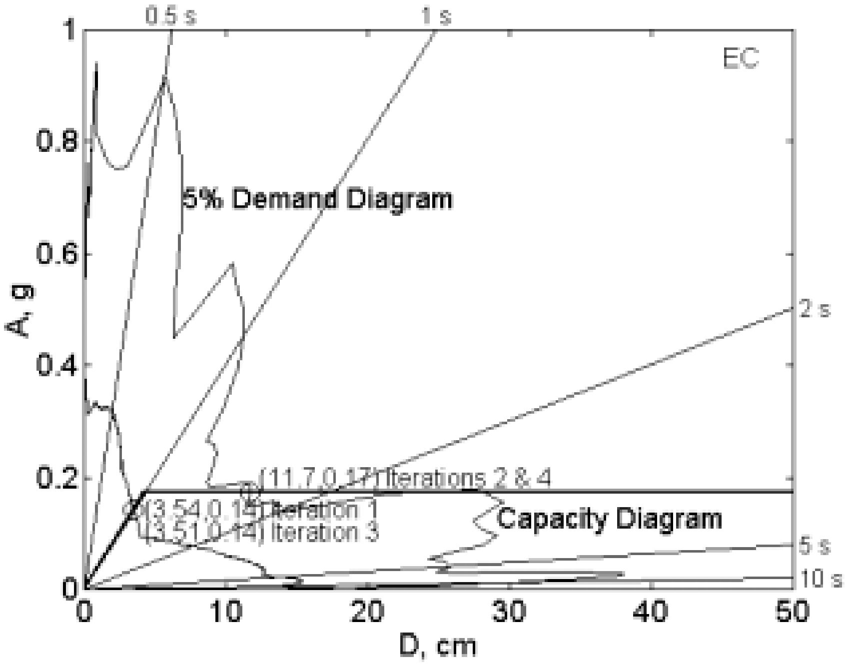
ATC-40 Procedure-A Analysis of System 5

- The ATC-40 Procedure A converges to a deformation much smaller than the “exact” value.
- Convergence is deceptive because it may leave the erroneous impression that calculated deformation is accurate.



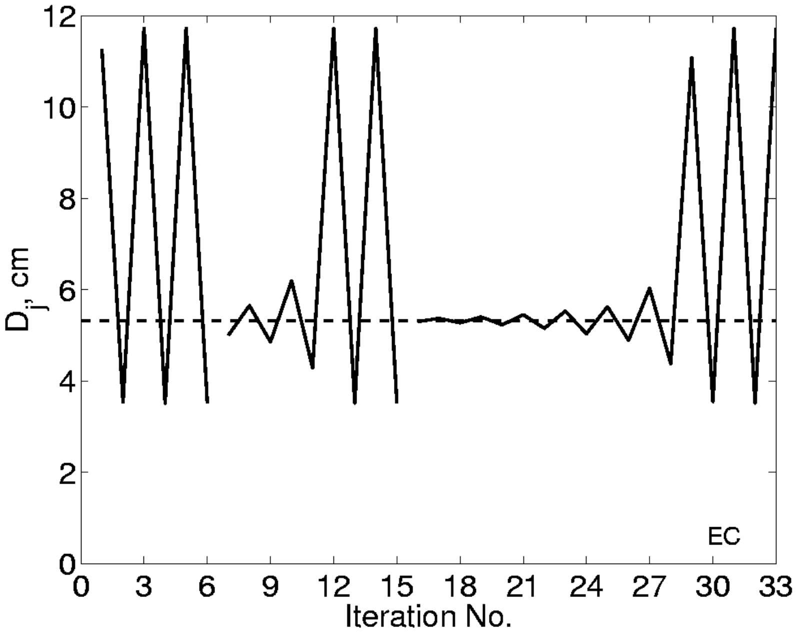
ATC-40 Procedure-A Analysis of System 6

- Given: $T_n = 1$ s, $\zeta = 0.05$, $f_y \div w = A_y \div g = 0.17$.
- Input: 1940 El Centro Ground Motion
- Find: D_{approx}
- Procedure-A fails to converge



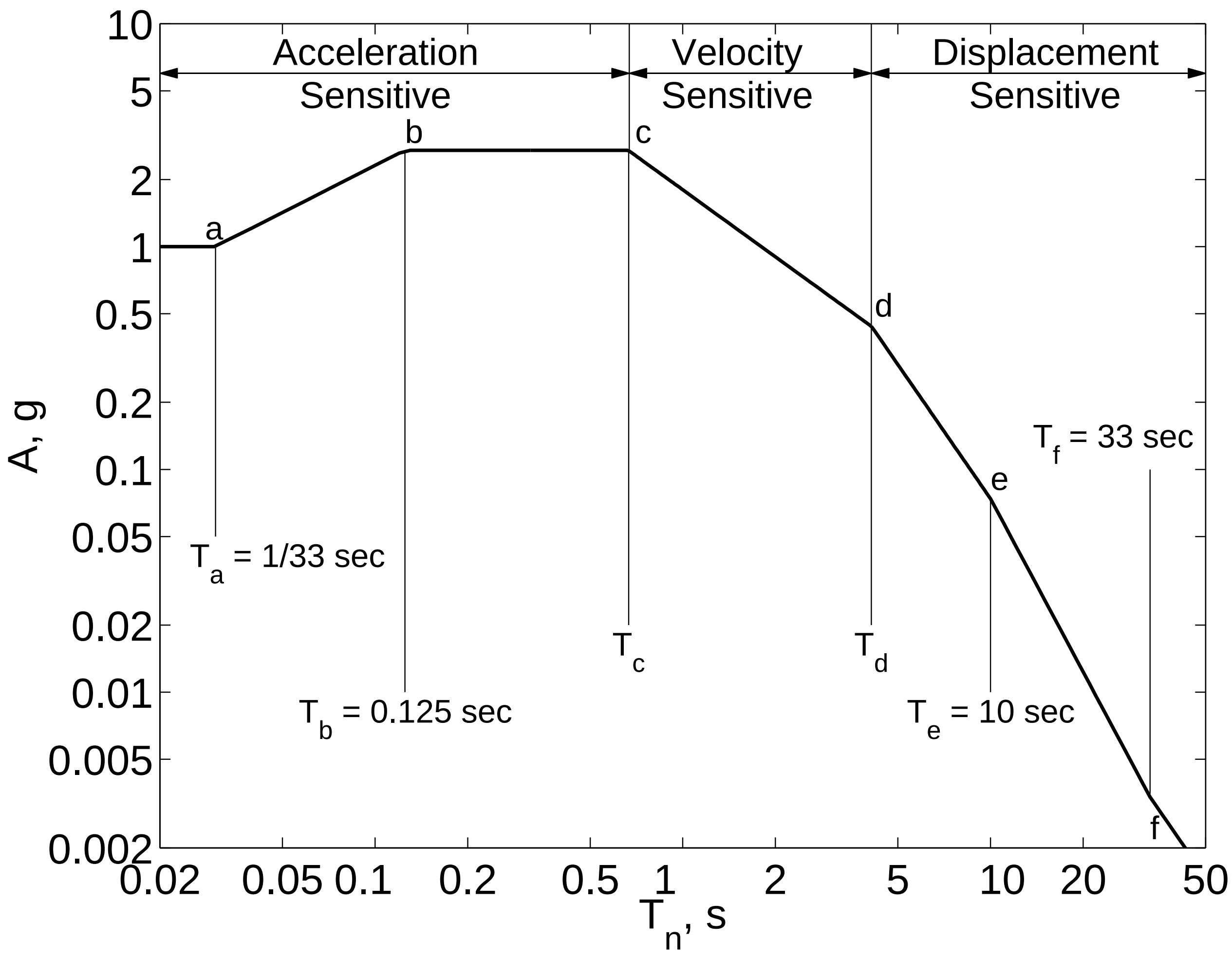
ATC-40 Procedure-A Analysis of System 6

- ATC-40 Procedure A fails to converge.
- Intersection point oscillates between 11.7 and 3.52 cm.
- Procedure diverges even when restarted with deformation close to “exact” value.



Examples: Design Spectrum

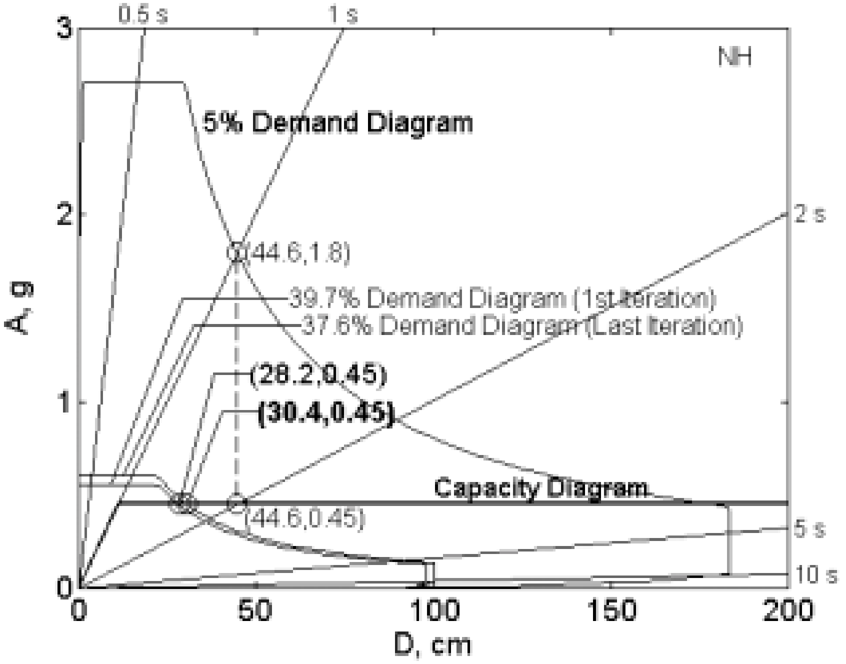
Newmark-Hall Elastic Design Spectrum



ATC-40 Procedure-A Analysis of System 5

- Given: T_n = 1 s, ζ = 0.05, $f_y \div w = A_y \div g$ = 0.45.
- Input: Newmark-Hall (1982) design spectrum
- Find: D_{approx}

1. Capacity and 5%-damped elastic demand diagram
2. Start at D =44.6 cm



ATC-40 Procedure-A Analysis of System 5

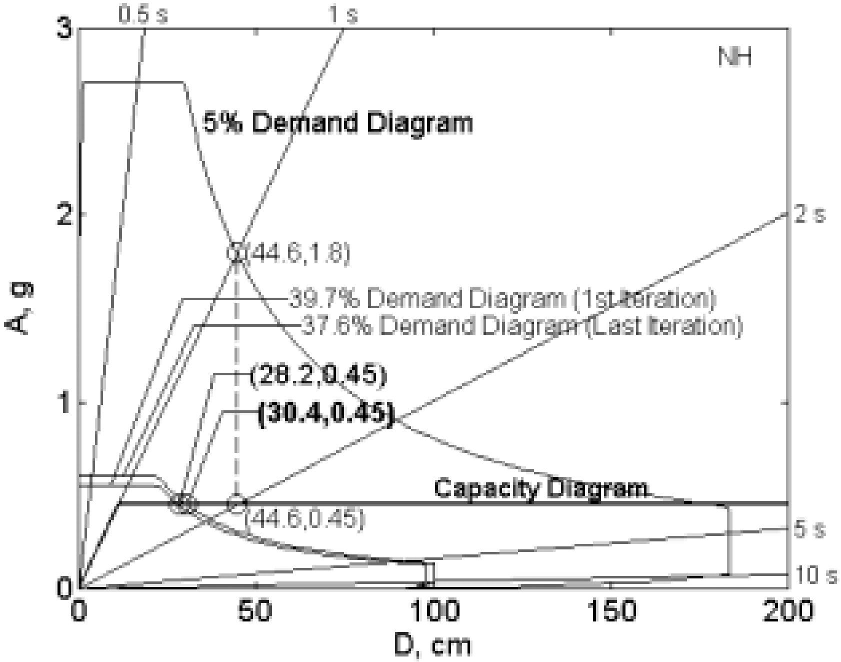
3. $\mu = 44.6 \div 11.6 = 4.0$

4. $\zeta_{eq} = (2/\pi) \times (\mu - 1/\mu) =$ ~~0.48~~ $= 0.45$

   $k = 0.77$

   $\zeta_{eq} = 0.05 + 0.77 \times 0.45 = 0.397$

5. Capacity diagram intersects 39.7%-damped elastic demand diagram at 28.2 cm



ATC-40 Procedure-A Analysis of System 5

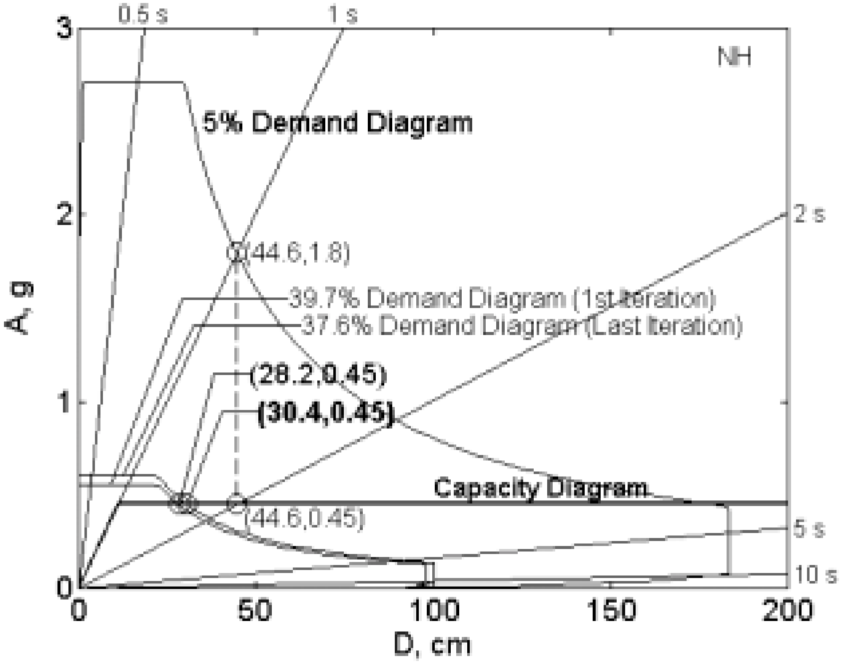
6. Error = 100×(28.2-44.6)÷44.6 = -58.4% > 5% tolerance

   Repeat steps 3 to 6 till convergence is achieved

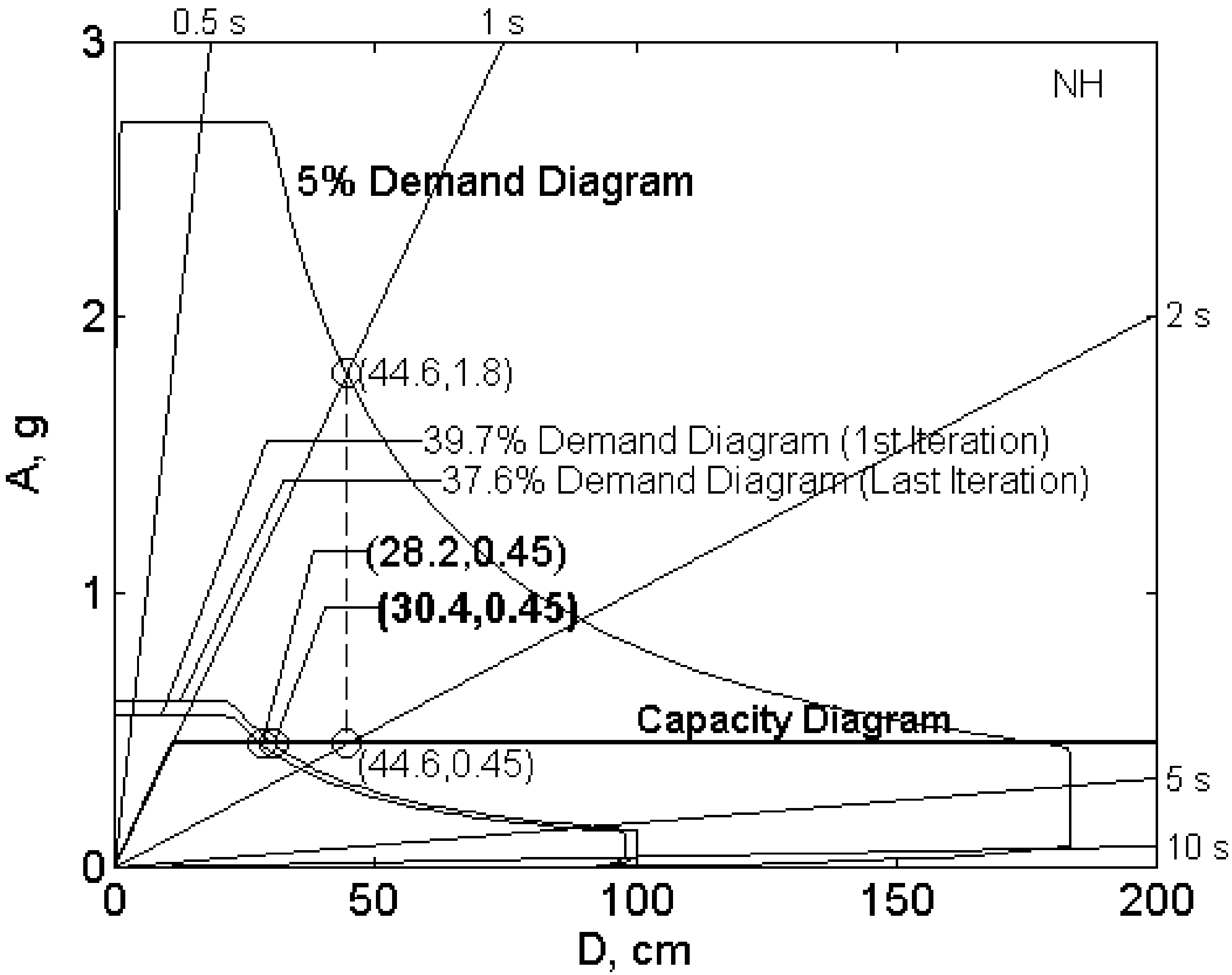
   D_{approx} = 30.4 cm

   D_{exact} = 44.6 cm

   Error = -31.8%

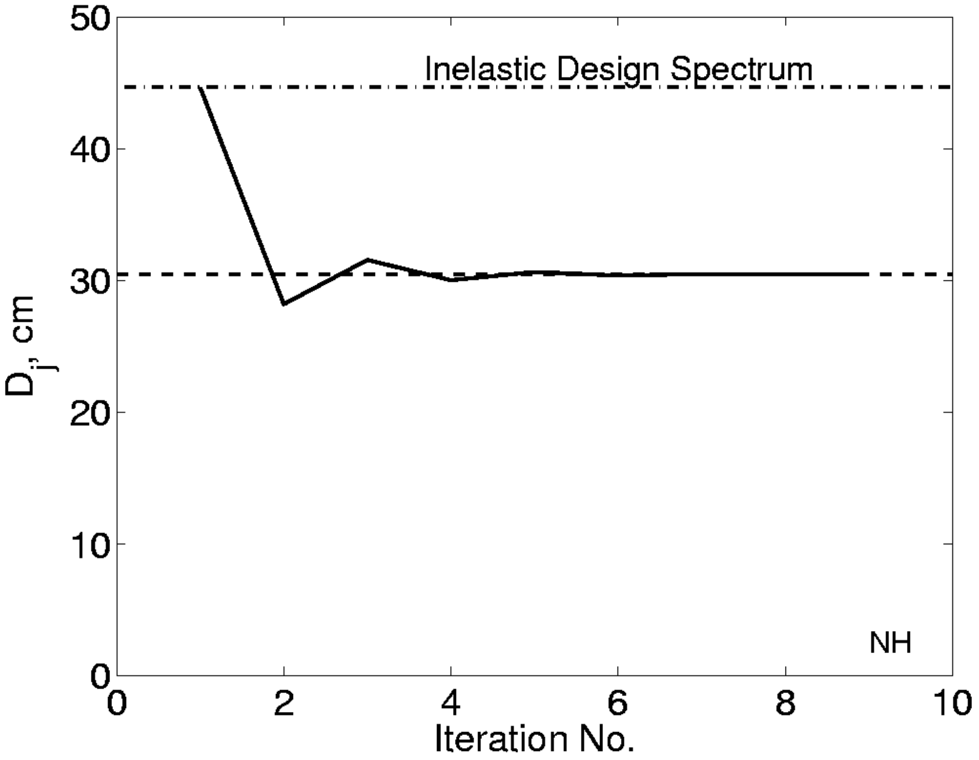


ATC-40 Procedure-A Analysis of System 5



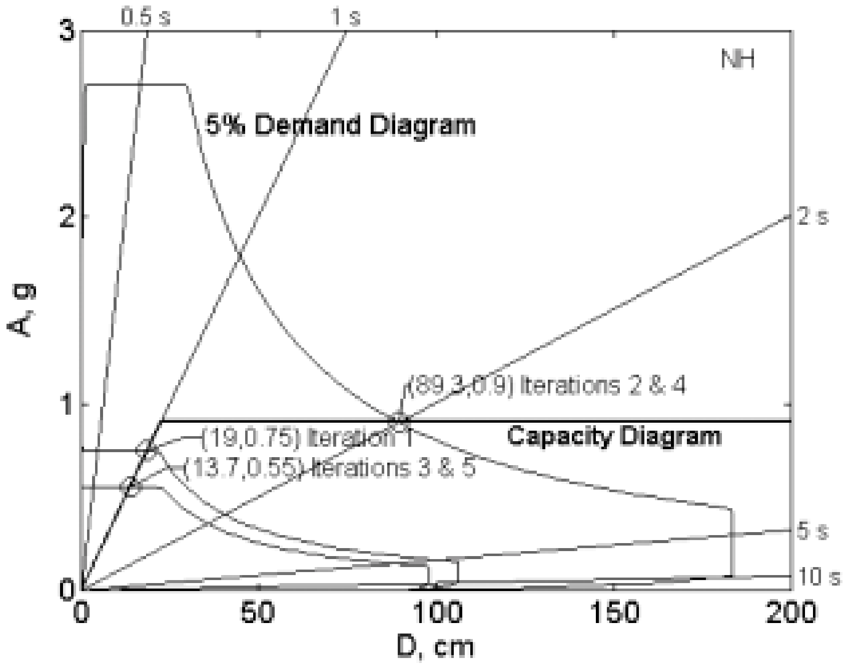
ATC-40 Procedure-A Analysis of System 5

- ATC-40 Procedure A gives deformation smaller than inelastic design spectrum.
- Convergence leaves an erroneous impression that calculated deformation is accurate.



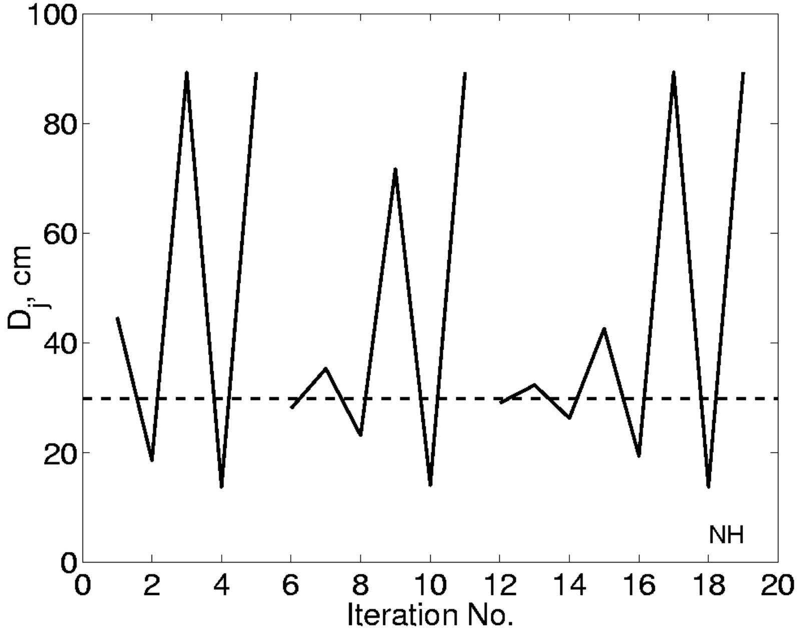
ATC-40 Procedure-A Analysis of System 6

- Given: T_n = 1 s, ζ = 0.05, $f_y \div w = A_y \div g$ = 0.90.
- Input: Newmark-Hall (1982) design spectrum
- Find: D_{approx}
- Procedure-A fails to converge



ATC-40 Procedure-A Analysis of System 6

- ATC-40 Procedure A fails to converge.
- Intersection point oscillates between 13.72 and 89.28 cm.
- Procedure diverges even when restarted with deformation close to value determined from inelastic design spectrum



ATC-40 Procedure B

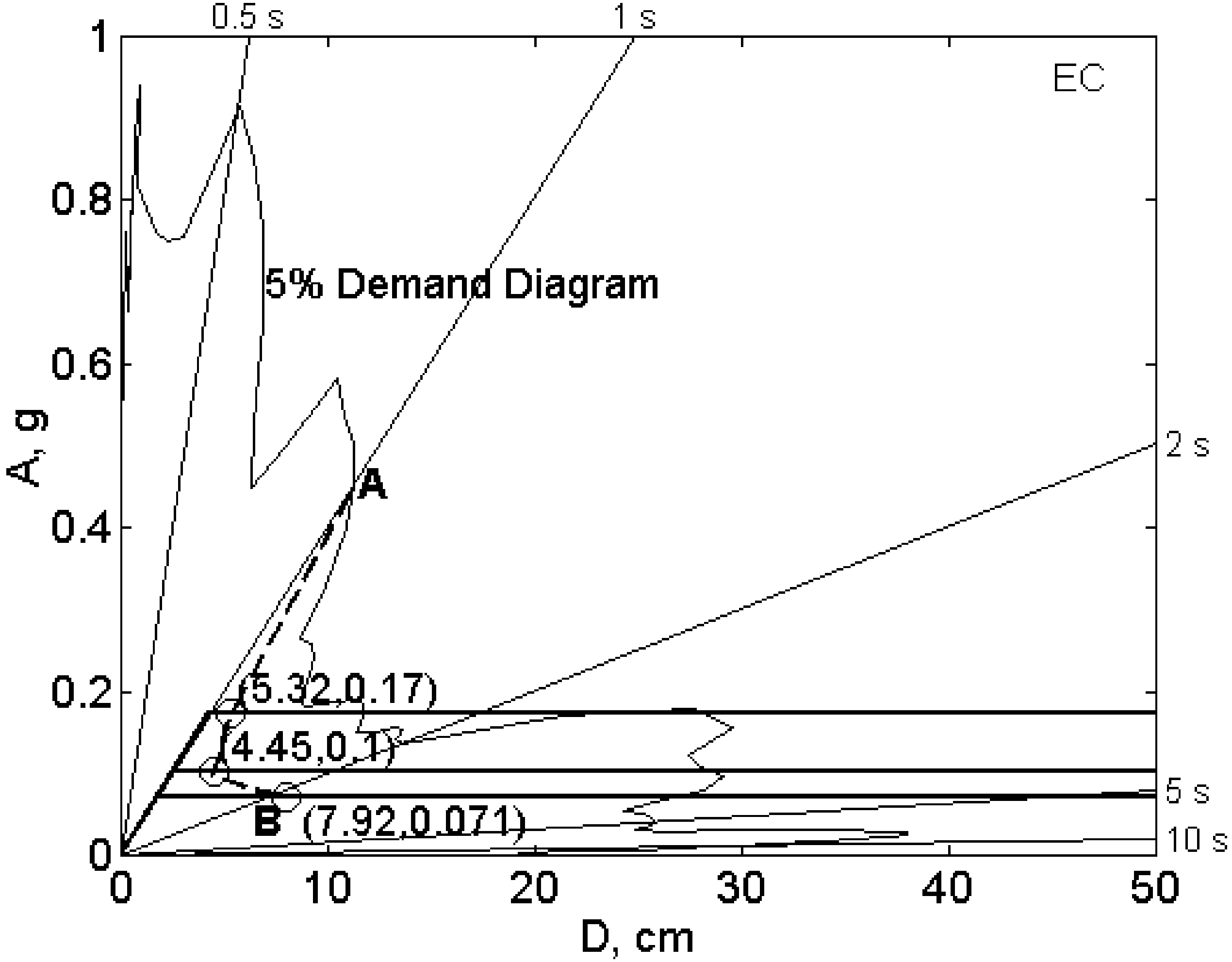
1. Plot the capacity diagram.
2. Estimate deformation demand D_i. Initially, assume $D_i = D(T_n, \zeta = 5\%)$.
3. Compute $\mu = D_i \div D_y$.
4. Compute T_{eq} and ζ_{eq}.
5. Compute $D(T_{eq}, \zeta_{eq})$ and $A(T_{eq}, \zeta_{eq})$ of an equivalent SDF system with T_{eq} and ζ_{eq}.
6. Plot $D(T_{eq}, \zeta_{eq})$ and $A(T_{eq}, \zeta_{eq})$.
7. Check if the curve generated by connecting points plotted in step 6 intersects the capacity diagram.

   If yes, intersection point gives D.

   Otherwise, repeat steps 3 to 7.

Examples: Ground Motion

ATC-40 Procedure-B Analysis of Systems 4 to 6



Application of ATC-40 Procedure B to Systems 5 and 6

- System 5

  D_{exact} = 10.16 cm

  D_{arrox} = 4.45 cm

  Error = -56.2%

- System 6

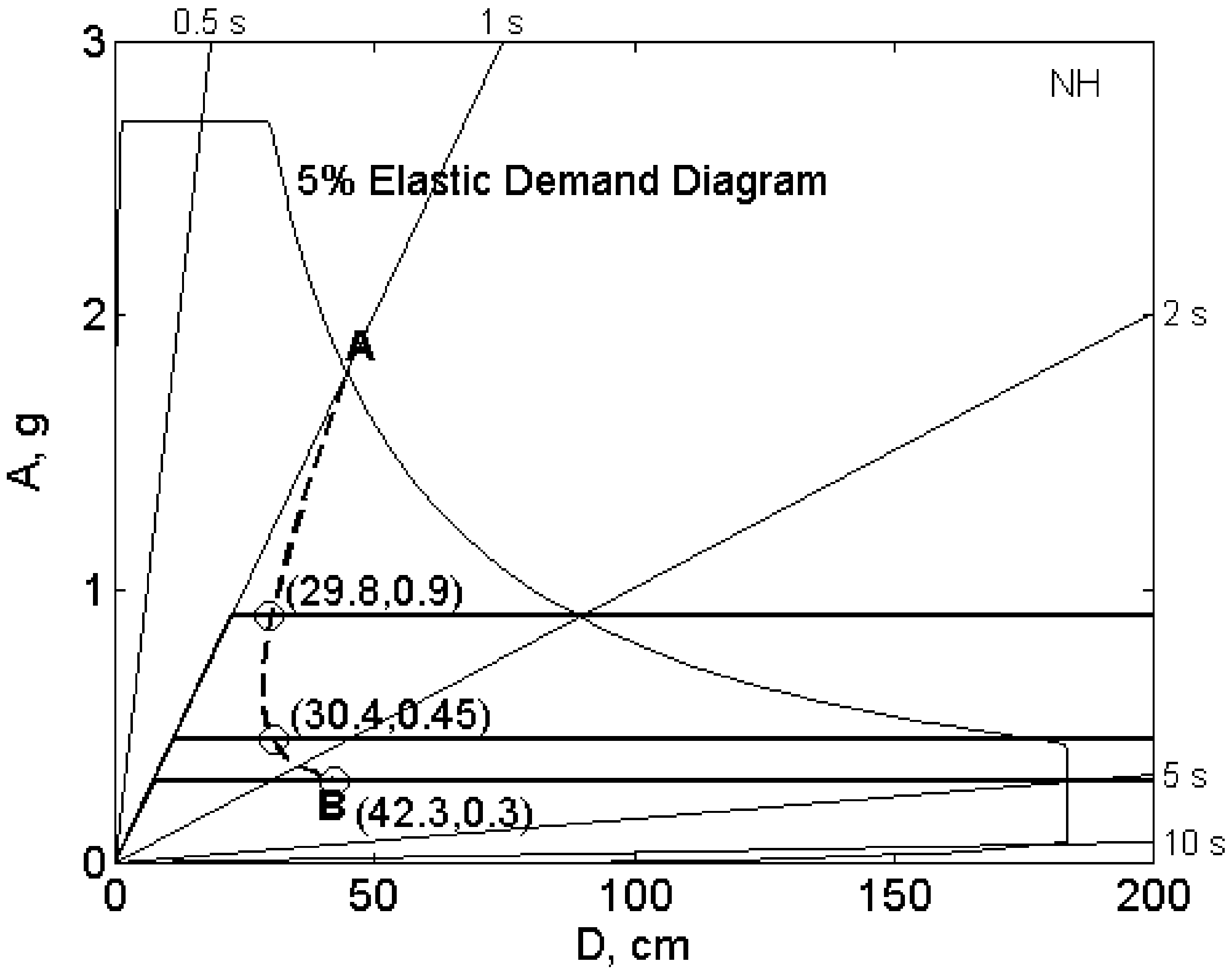
  D_{exact} = 8.53 cm

  D_{arrox} = 5.32 cm

  Error = -37.7%

Examples: Design Spectrum

ATC Procedure-B Analysis of Systems 4 to 6



ATC-40 Procedure-B Analysis of Systems 5 and 6

- System 5

  D_{exact} = 44.6 cm

  D_{arrox} = 30.4 cm

  Error = -31.7%

- System 6

  D_{exact} = 44.6 cm

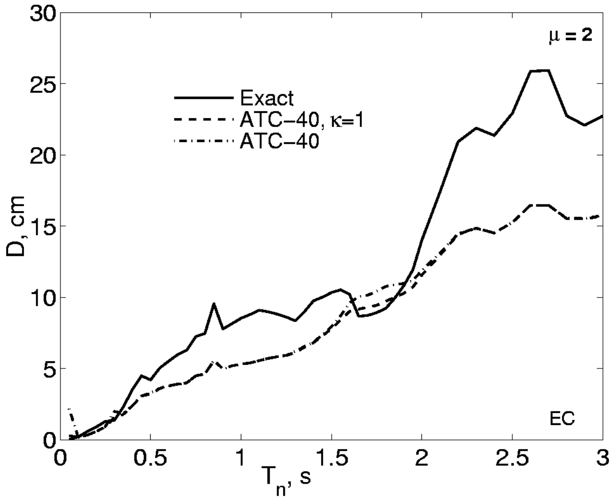
  D_{arrox} = 29.8 cm

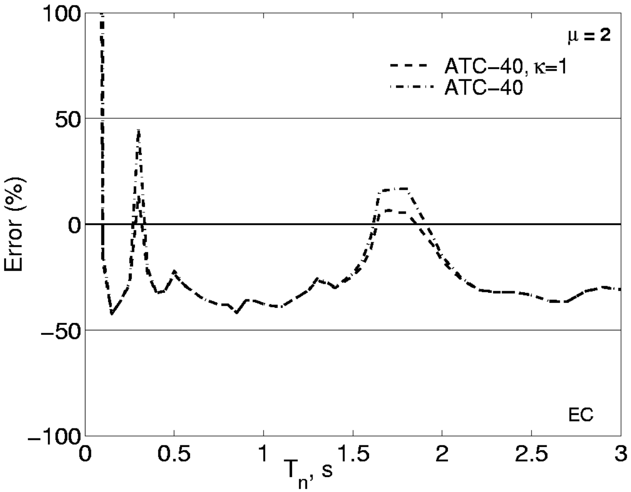
  Error = -33.1%

Evaluation of ATC-40 Procedure: Ground Motion

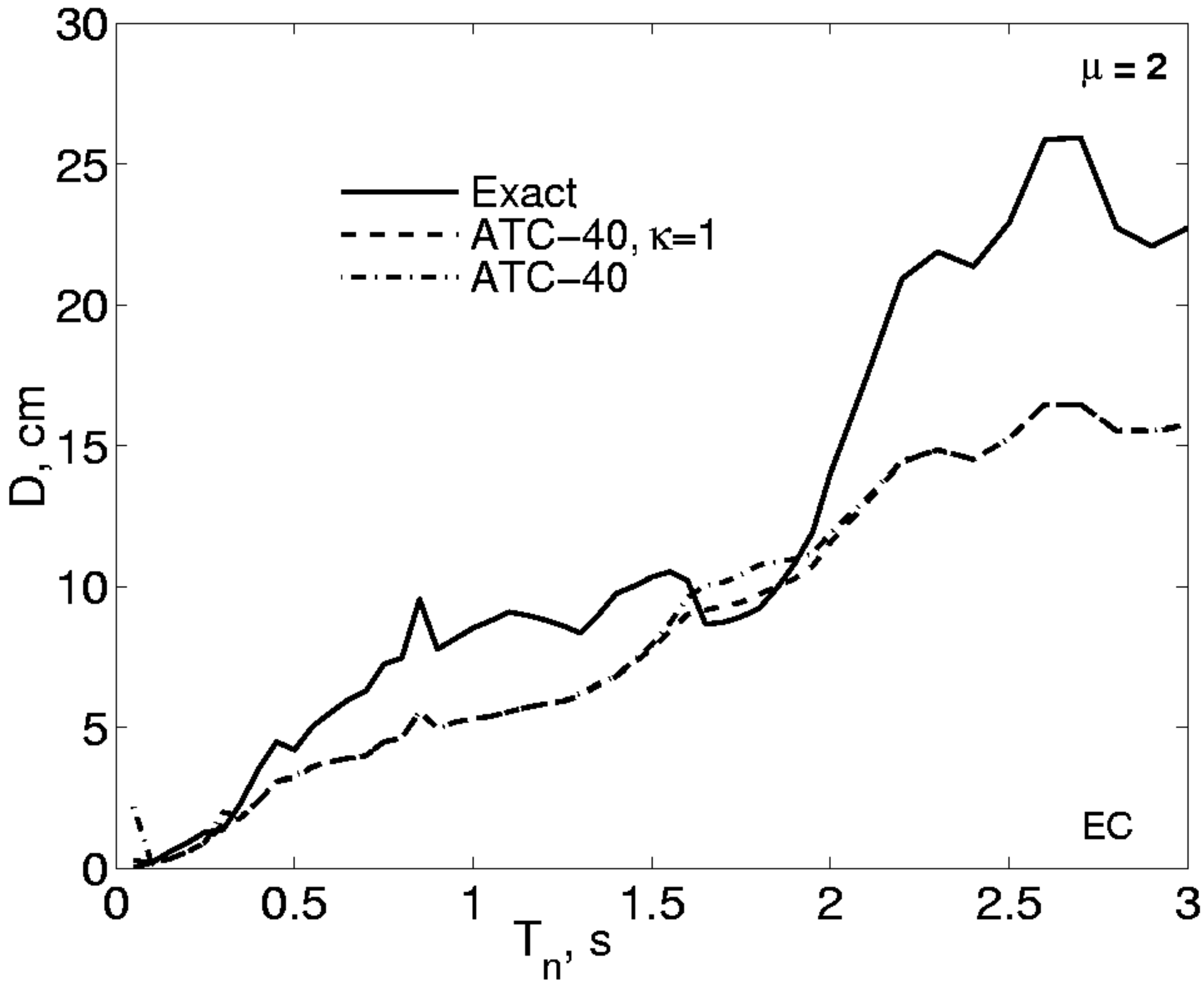
Comparison of deformations: $\mu = 2$

- ATC-40 method is inaccurate
  - ➔ Underestimates deformation significantly over a wide period range
  - ➔ $D \cong$ Half of "Exact"
- Damping modification factor, κ, is not attractive
  - ➔ Results improve marginally
  - ➔ κ based on judgement

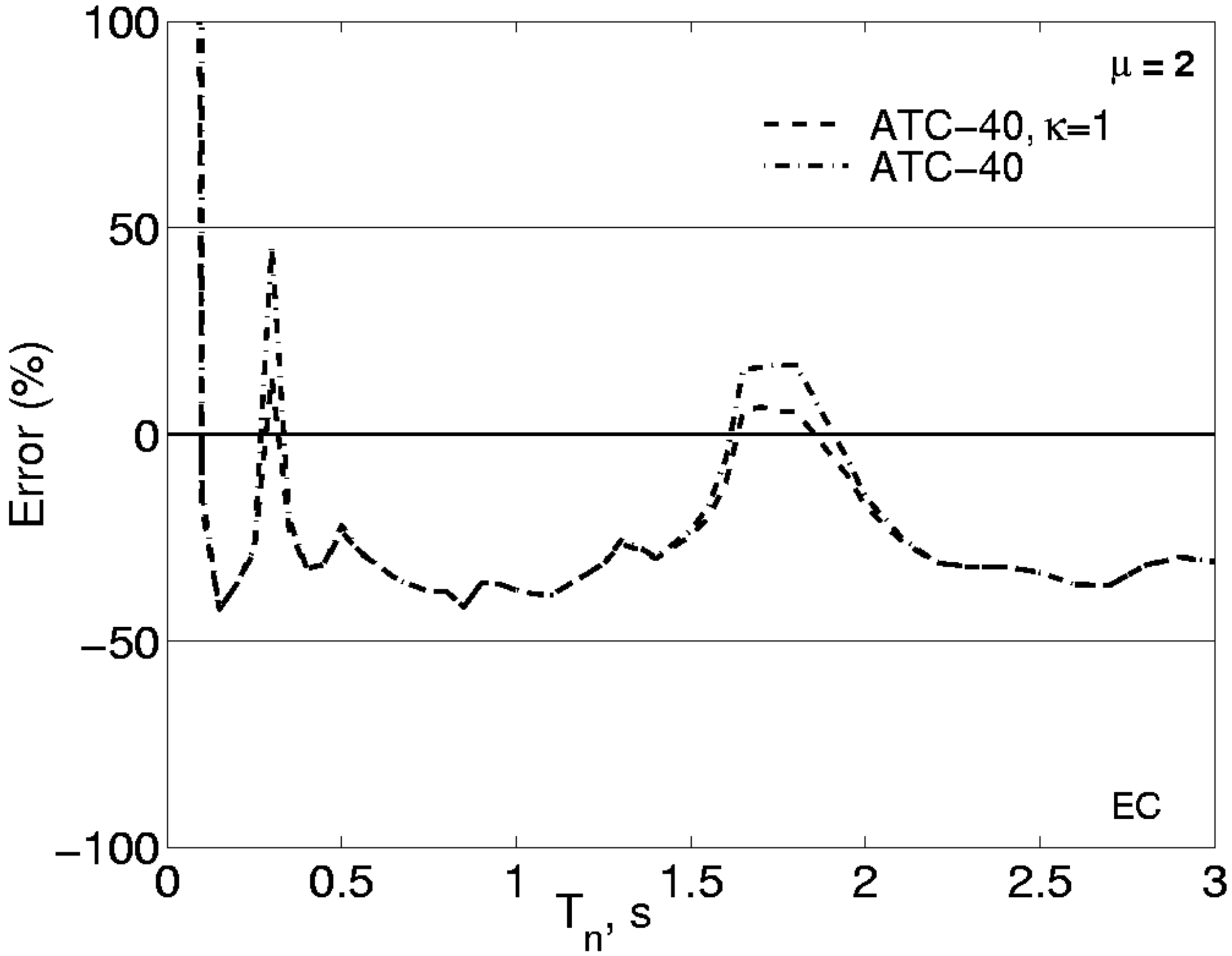




Comparison of deformations: $\mu = 2$

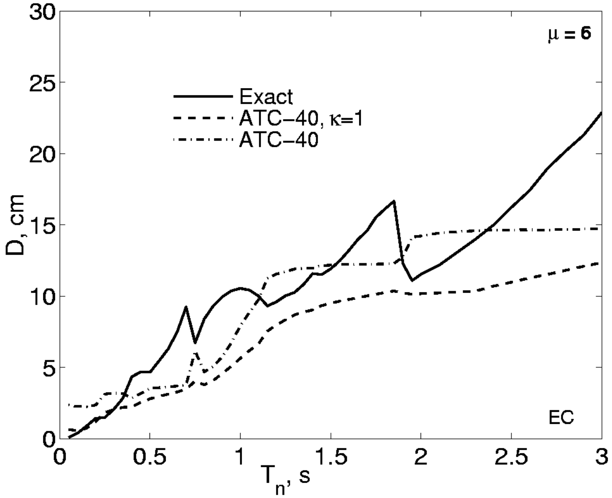


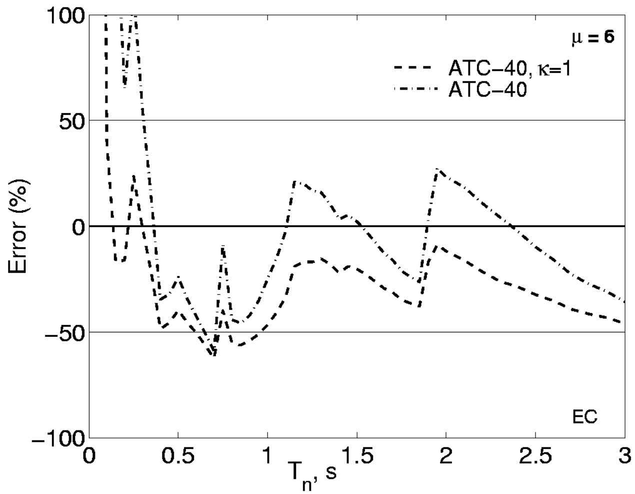
Errors in deformations: $\mu = 2$



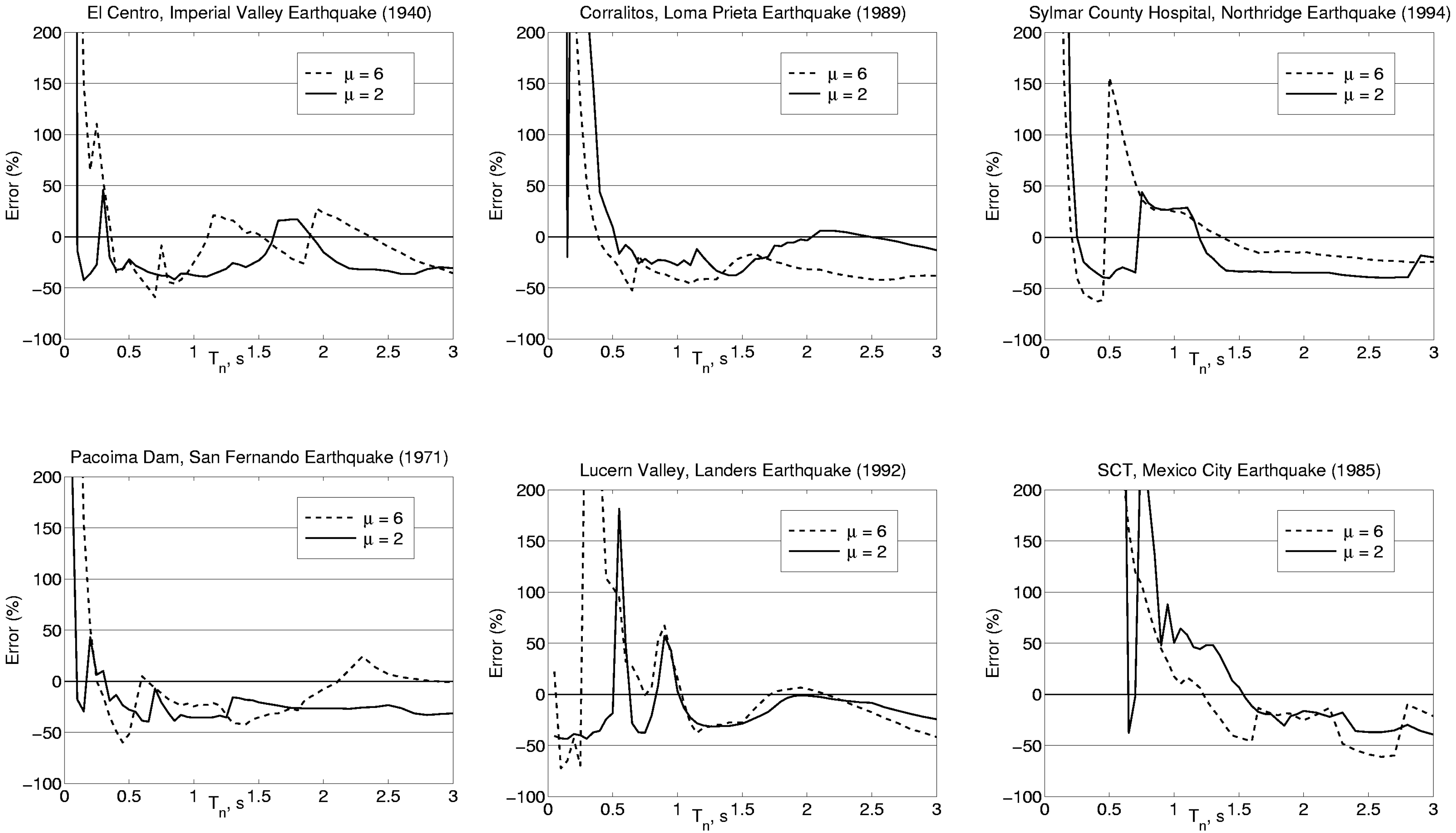
Comparison of deformations: $\mu = 6$

- ATC-40 method is inaccurate
  - ➔ Underestimates deformation significantly over a wide period range
- Damping modification factor, κ, is not attractive
  - ➔ Results improve marginally
  - ➔ κ based on judgement





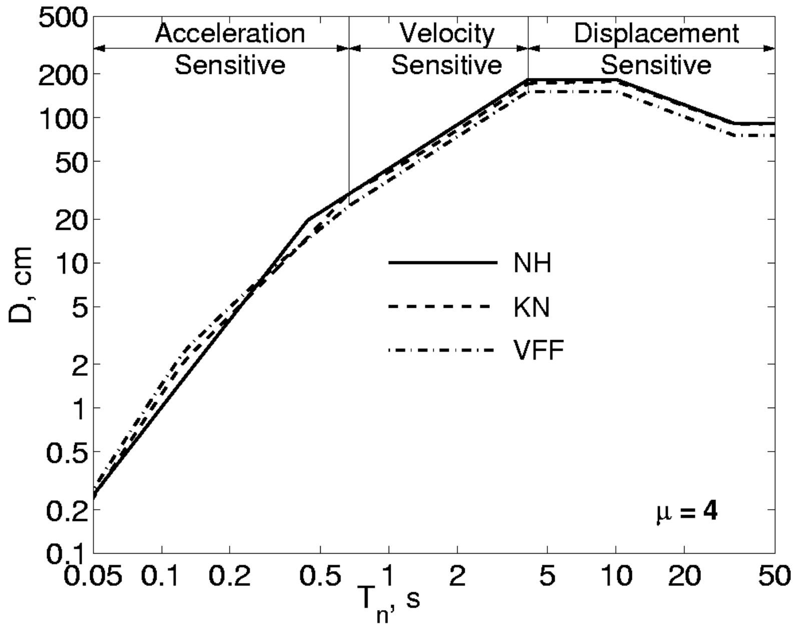
Errors for Six Ground Motions



Evaluation of ATC-40 Procedure: Design Spectrum

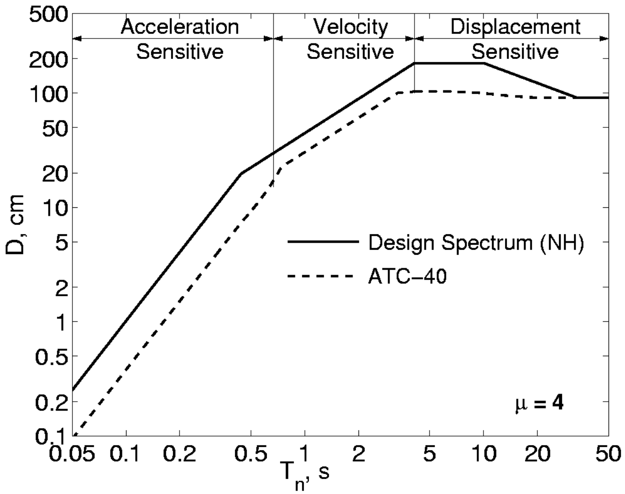
Deformation Spectra for Inelastic Systems

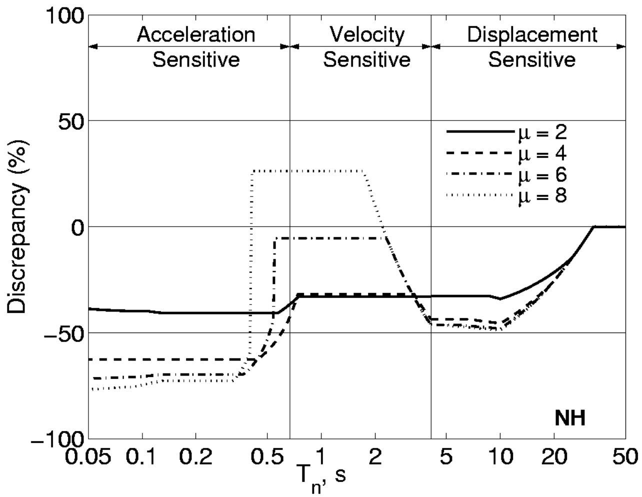
- Newmark-Hall (NH)
- Krawinkler-Nassar (KN)
- Vidic, Fajfar, and Fischinger (VFF)
- NH and KN give similar results except for $T_n < 0.3$ sec



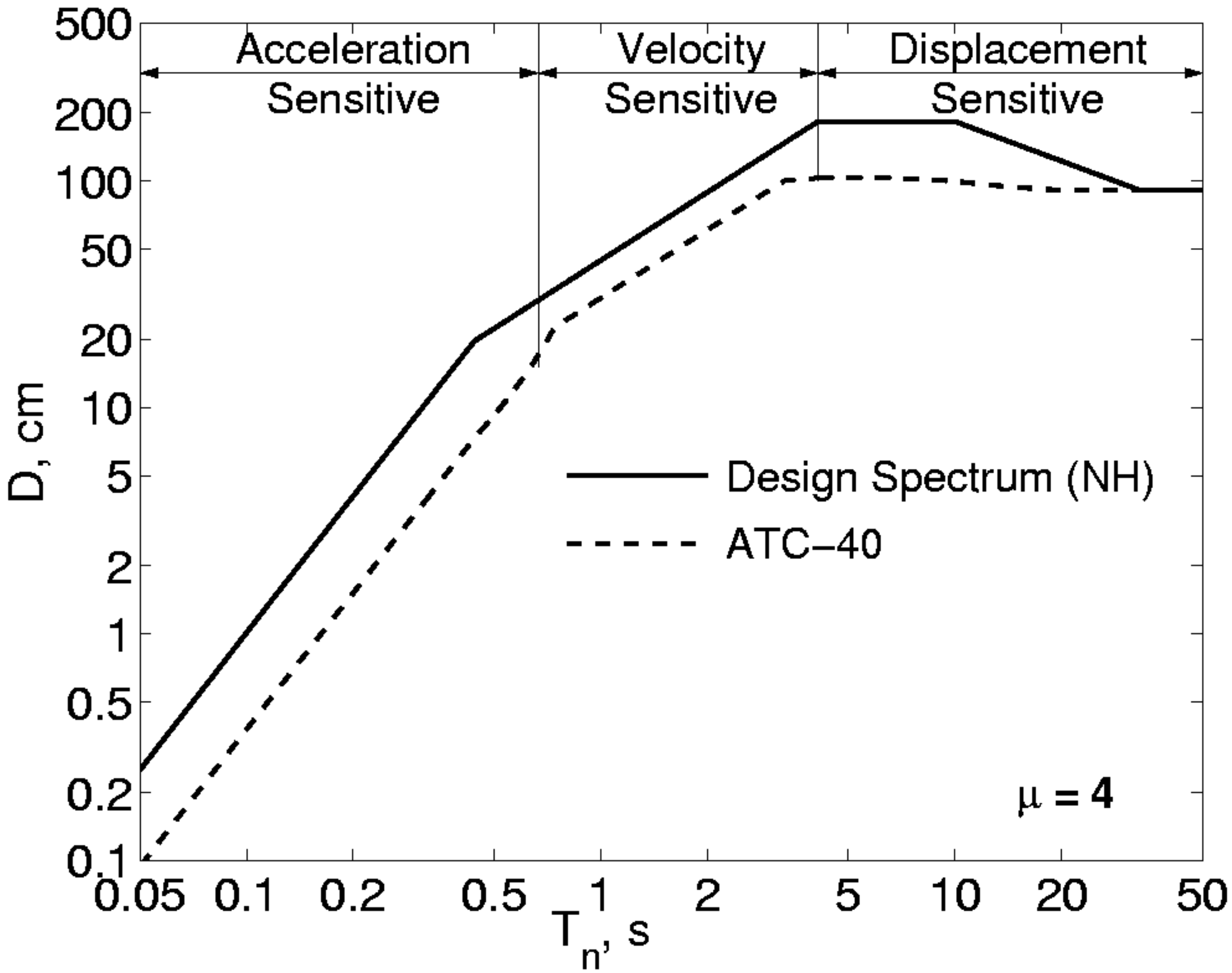
Comparison of Deformations (NH)

- Large errors in ATC-40
- Discrepancy depends on ductility and period region
  - ➔ Underestimates deformation in acc. and disp. regions; discrepancy increases with increasing μ
  - ➔ Underestimates deformation in velo. region for $\mu = 2$ and 4, but overestimates for $\mu = 8$.
- ATC-40 method is deficient compared to **elastic** spectrum in velo. and disp. regions

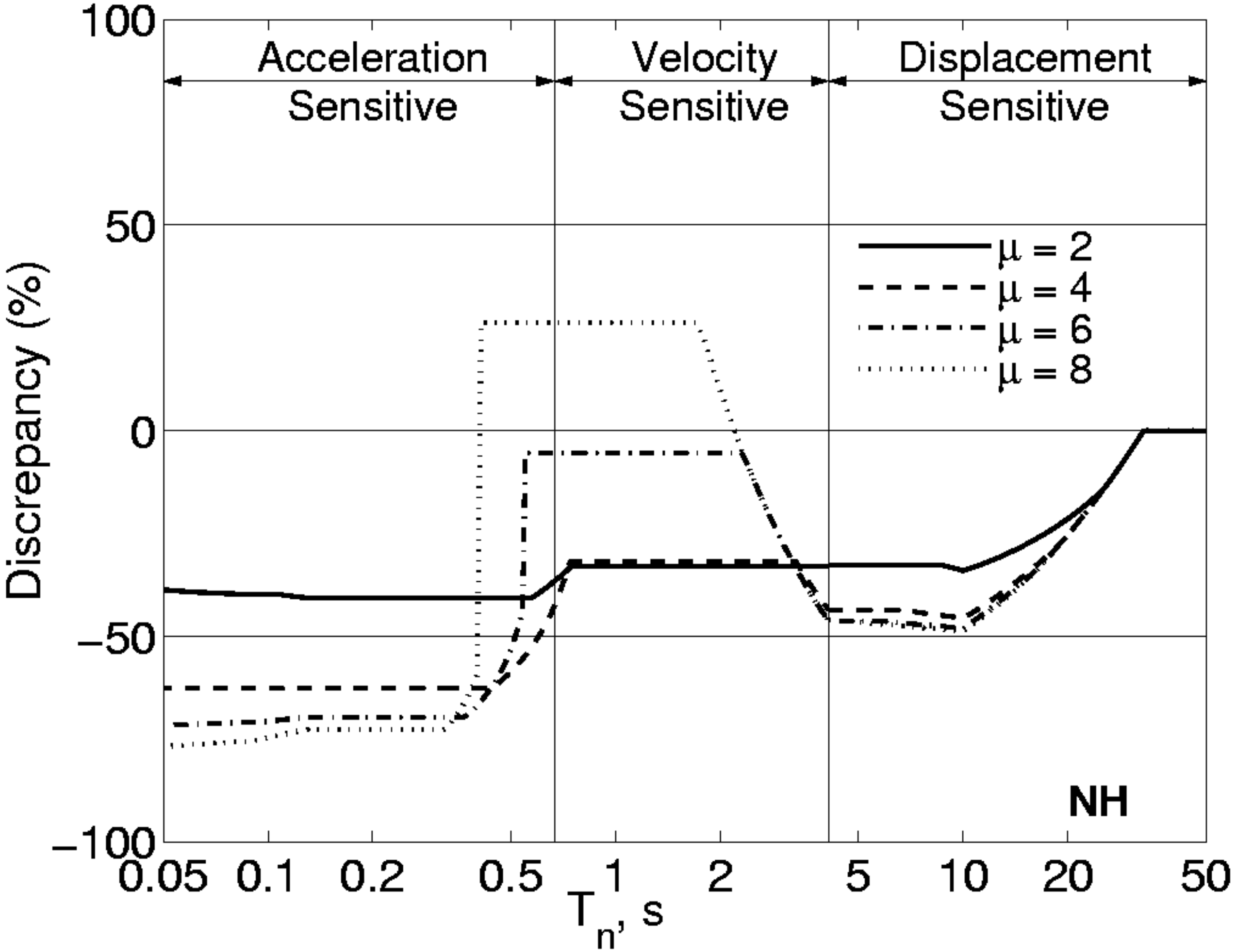




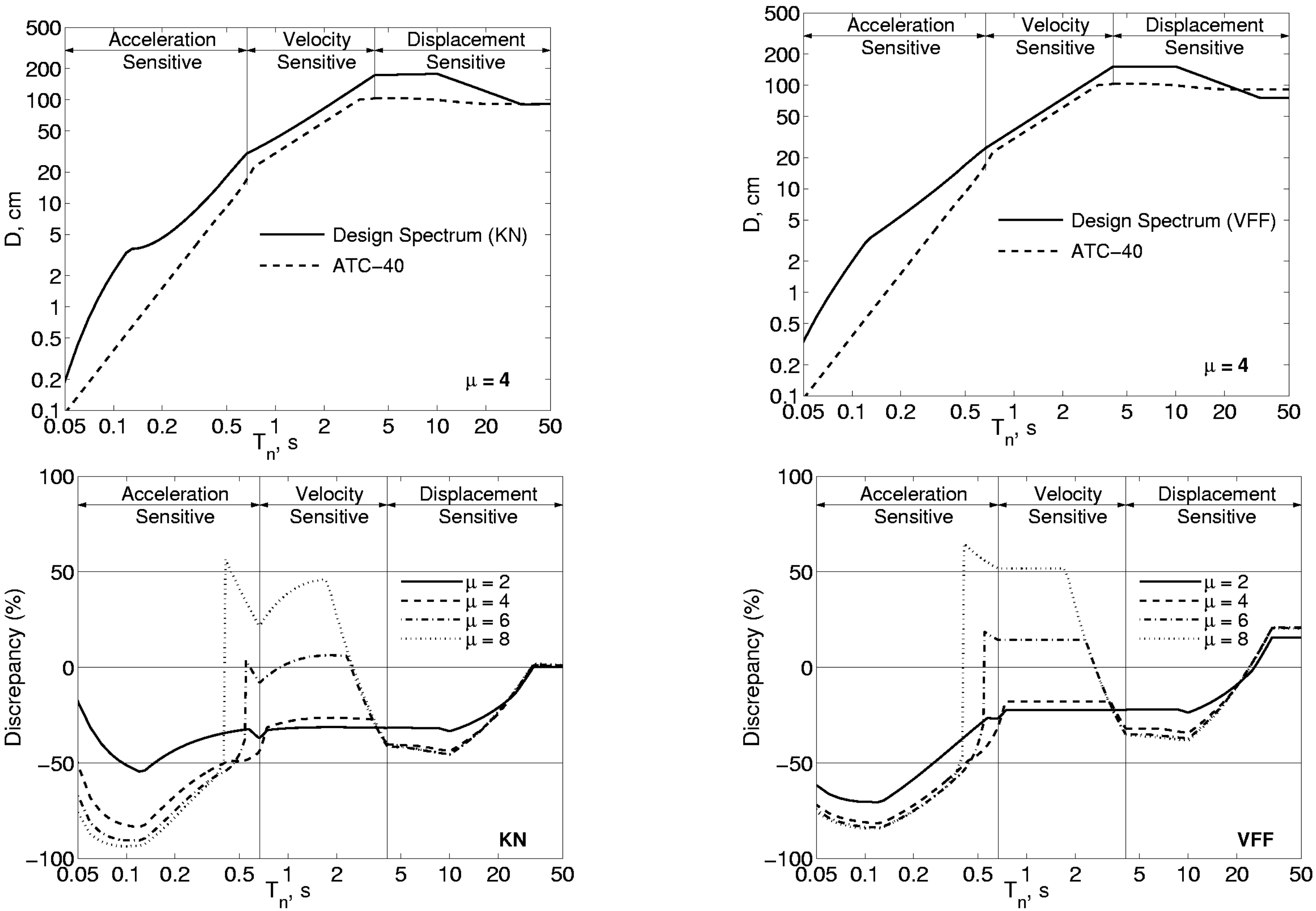
Comparison of Deformations (NH)



Errors in Deformations (NH)



Results for NH, KN and VFF Design Spectra



Improved Procedures

- Existing procedures use equivalent linear systems
  - ➔ Convergence of iterative procedure?
  - ➔ Excessive damping?
  - ➔ Large errors
- Develop procedures based on inelastic design spectrum
  - ➔ Eliminate errors (or discrepancies)
  - ➔ Retain graphical appeal

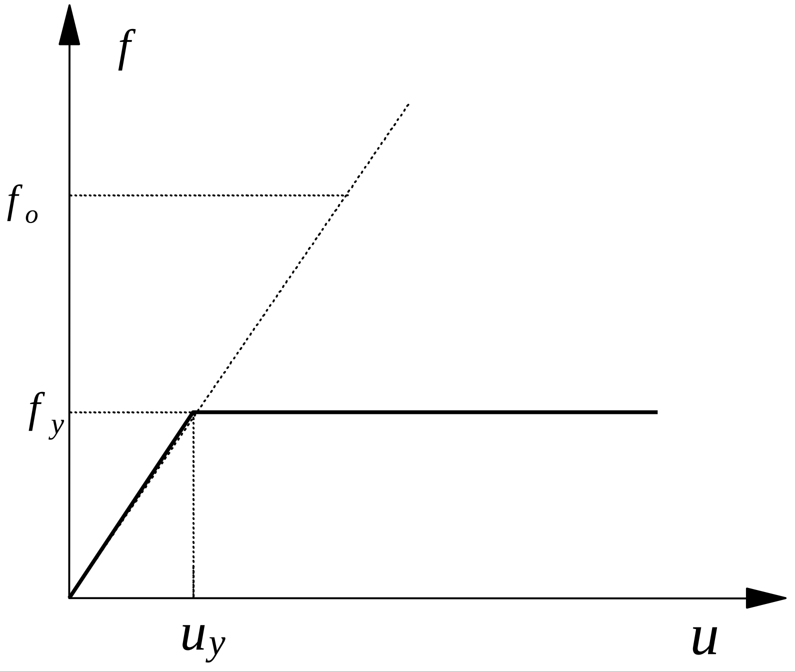
Definitions

- f_o = Strength demand for elastic behavior
- f_y = Yield strength
- Normalized yield strength

$$\bar{f}_y = \frac{f_y}{f_o}$$

- Yield strength reduction factor

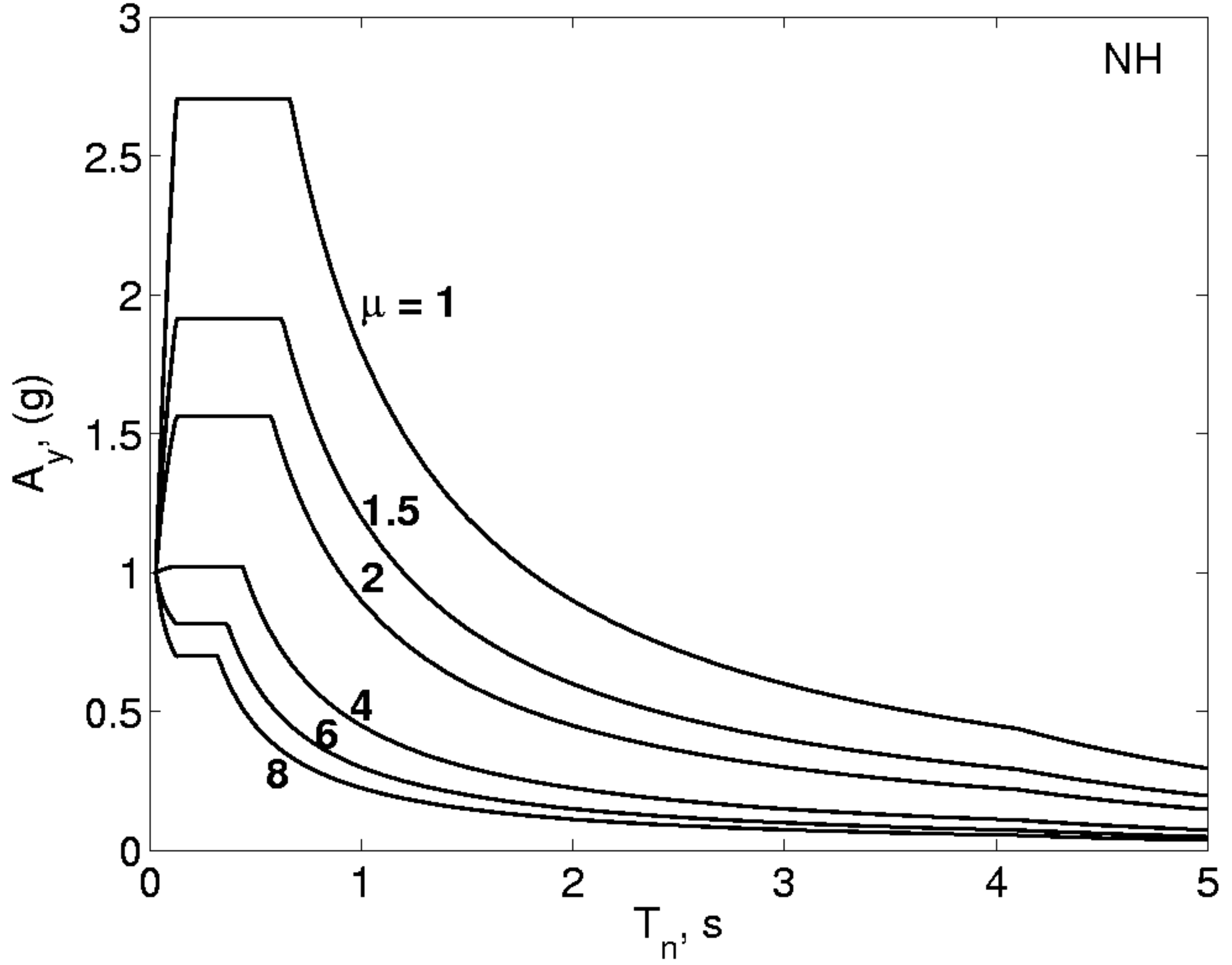
$$R_y = \frac{f_o}{f_y}$$



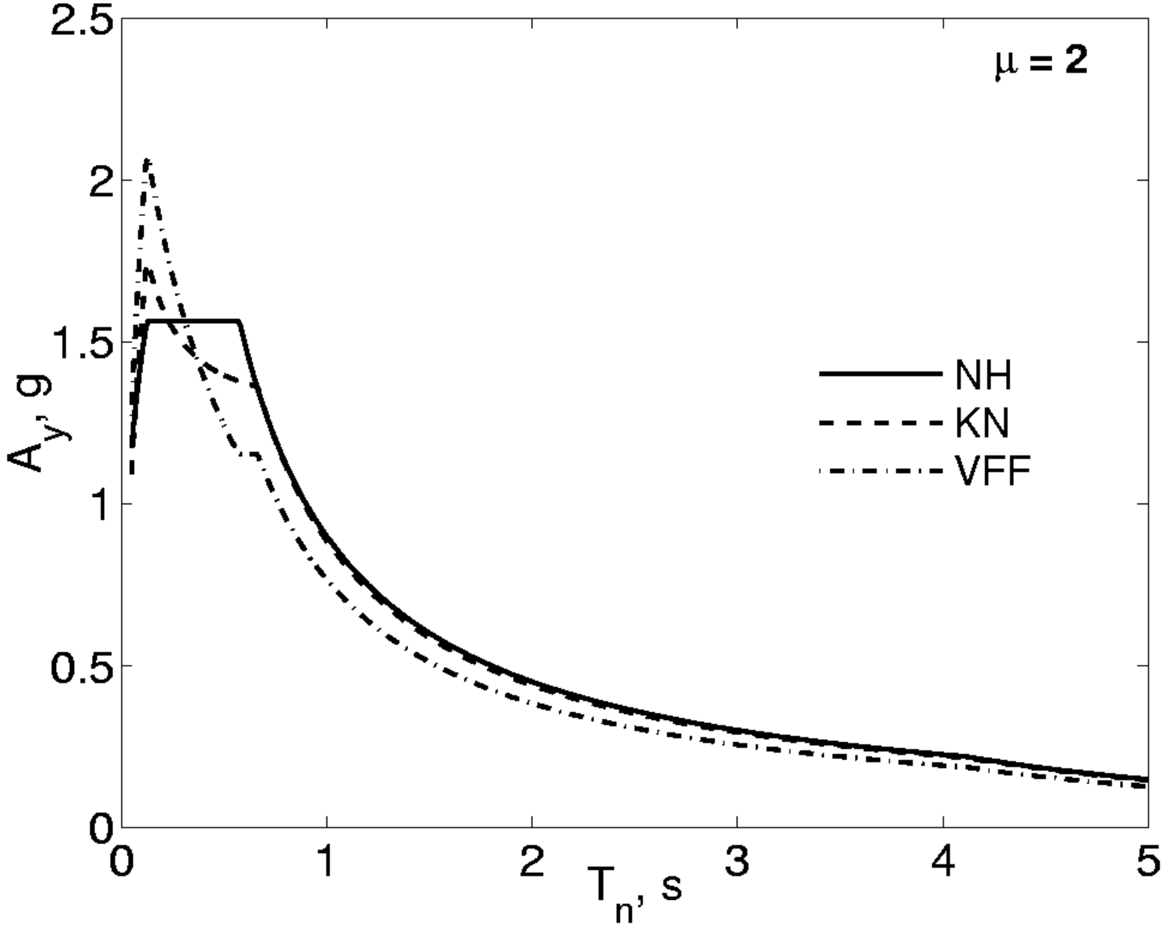
Inelastic Design Spectrum

- Yield strength required for structure to remain elastic $f_o = \frac{A}{g} w$
- Yield strength of inelastic structure $f_y = \frac{A_y}{g} w$
- Yield strength reduction factor: $R_y(\mu, T_n)$ $A_y = \frac{A}{R_y}$
- Plot of A_y v's T_n
- R_y–μ–T_n relations
  - ➔ Newmark-Hall (1982)
  - ➔ Krawinkler-Nassar (1992)
  - ➔ Vidic-Fajfar-Fischinger (1994)
  - ➔ Others

Inelastic Design Spectra

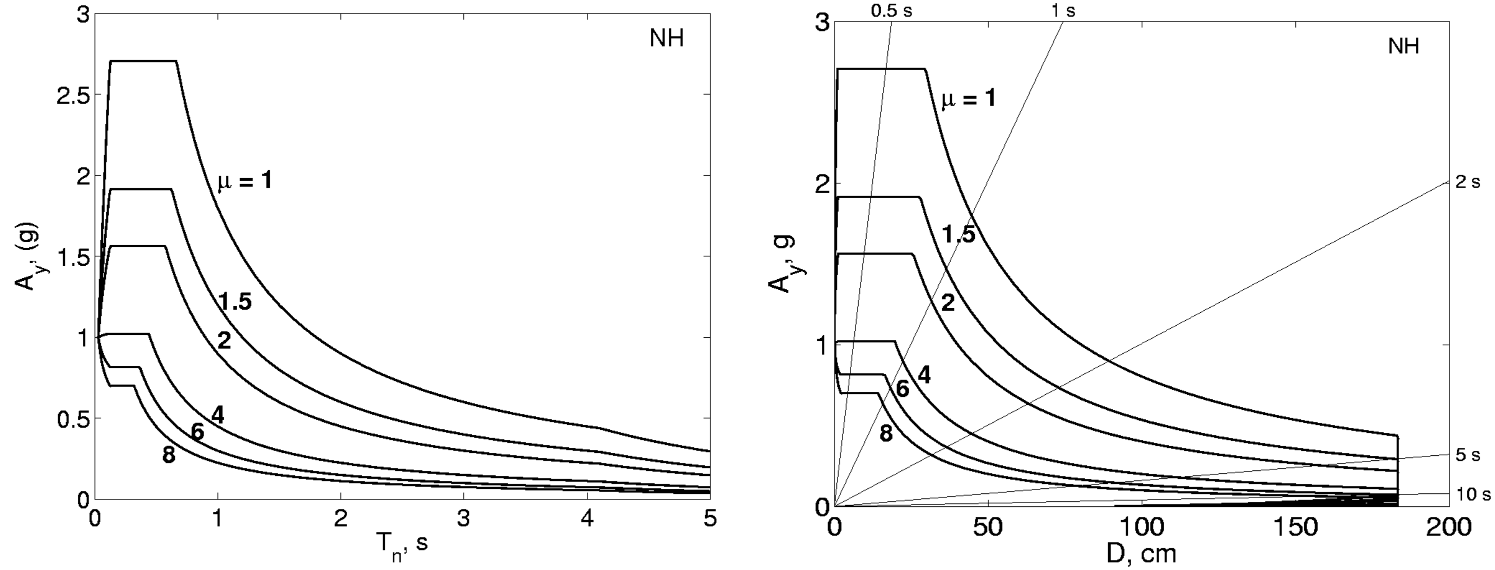


Inelastic Design Spectra Using Three Different R_y–μ–T_n Equations



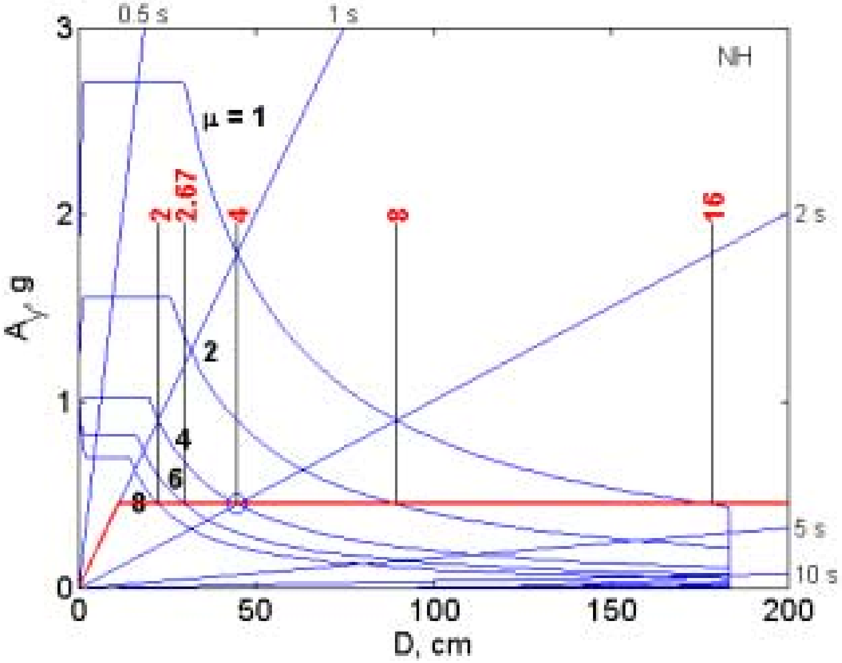
Inelastic Demand Diagram

- Inelastic design spectrum plotted in *A-D* format
- Deformation from spectrum: $D = m\,D_y = \mu(T_n \div 2\pi)^2 A_y$
- Plot A_y v's D for constant μ

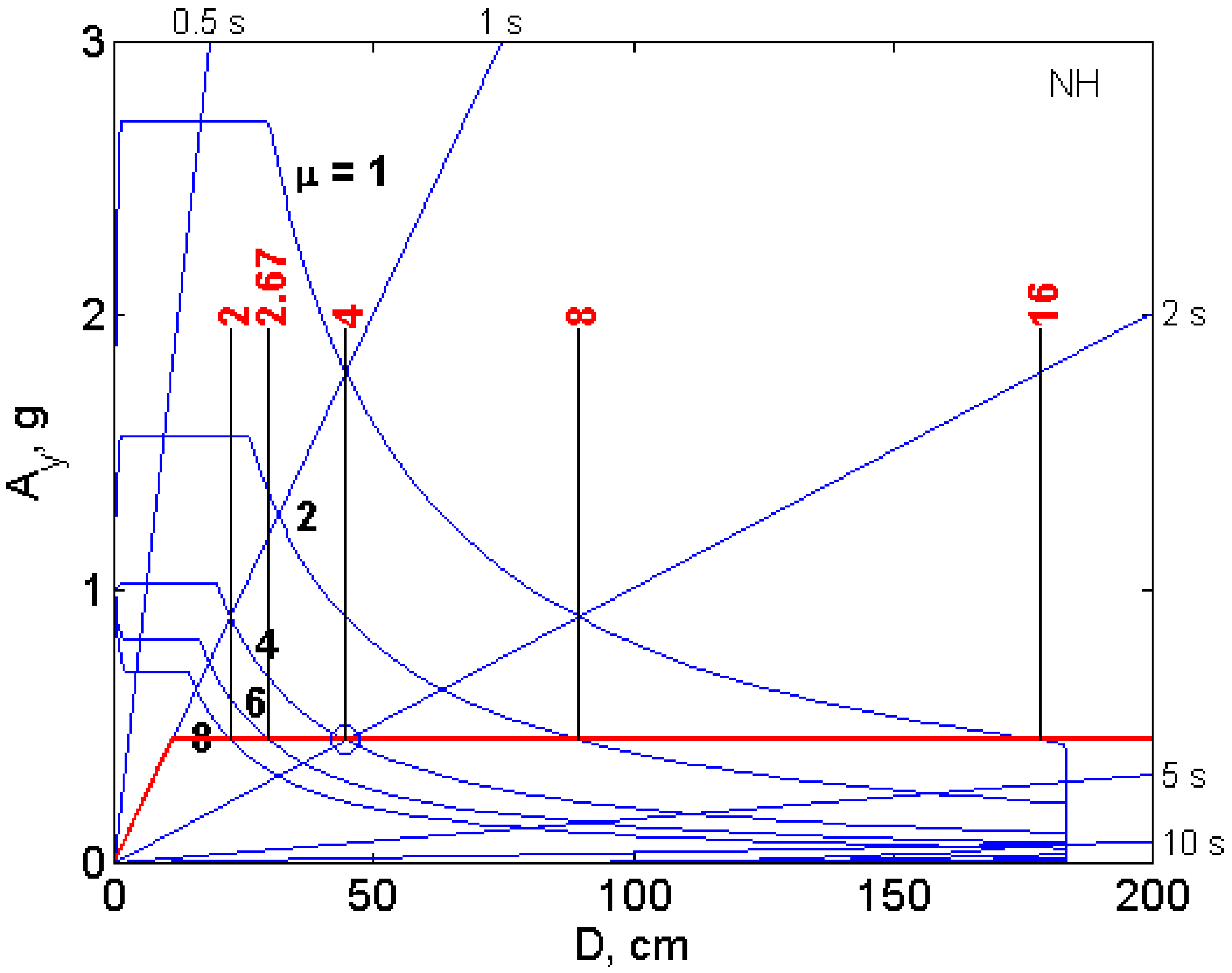


Improved Procedure-A Analysis of System 5

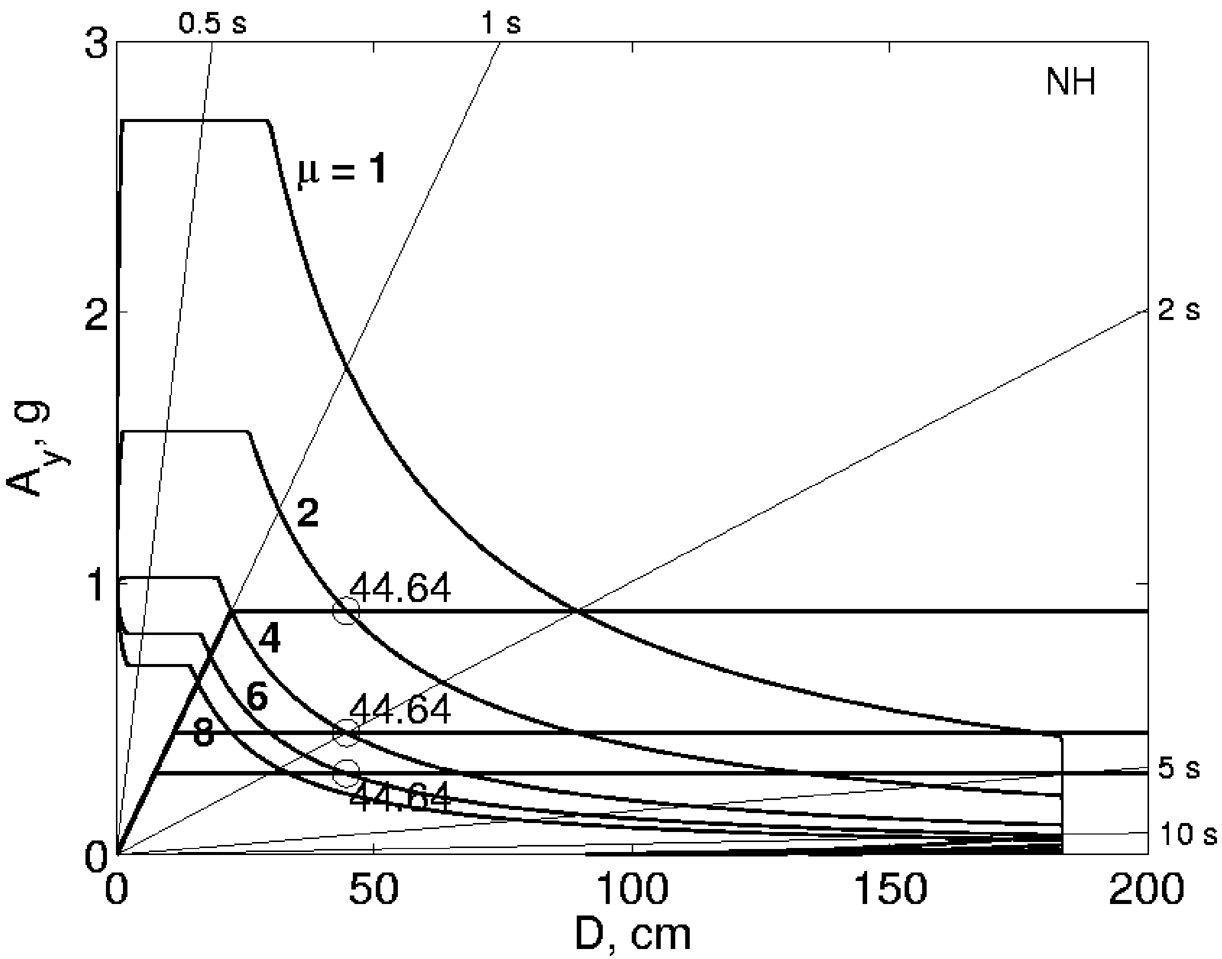
- Plot capacity and demand diagrams in *A-D* format
- Yielding branch of capacity diagram intersects the demand diagram for several μ
- **At relevant intersection point, μ from the two diagrams should match**
- Interpolate between two μ values or plot demand diagrams at finer μ values if necessary



Improved Procedure-A Analysis of System 5

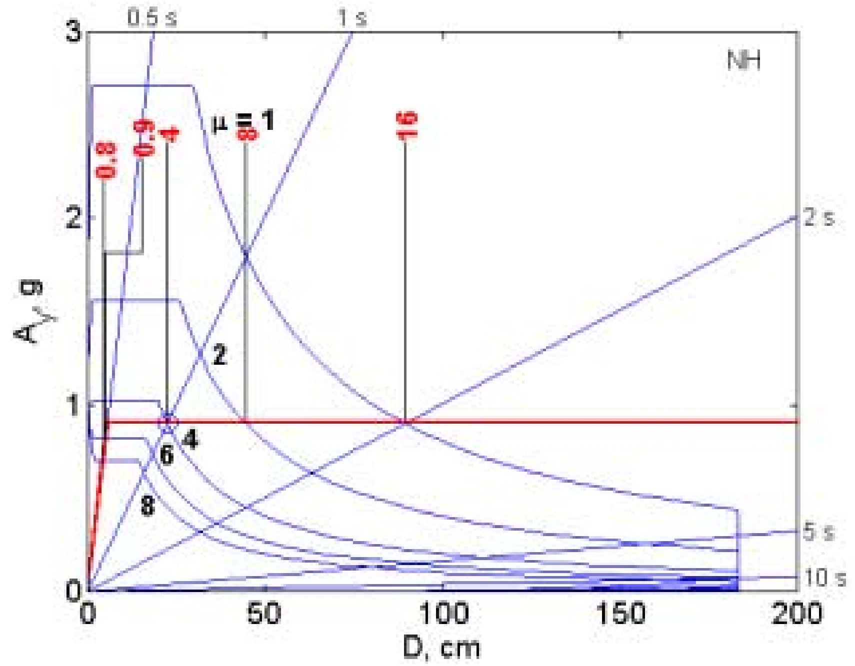


Improved Procedure-A Analysis of Systems 4 to 6

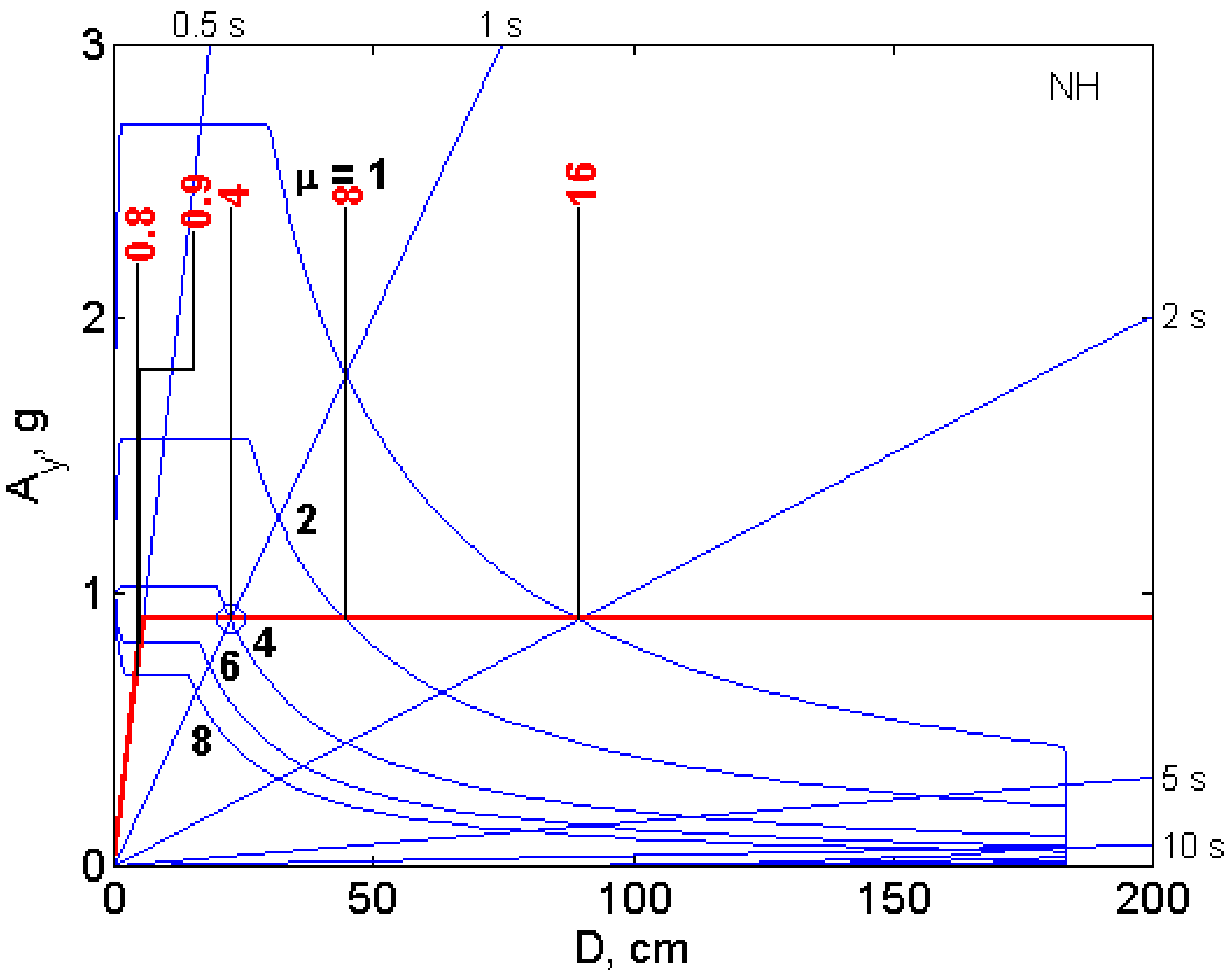


Improved Procedure-A Analysis of System 2

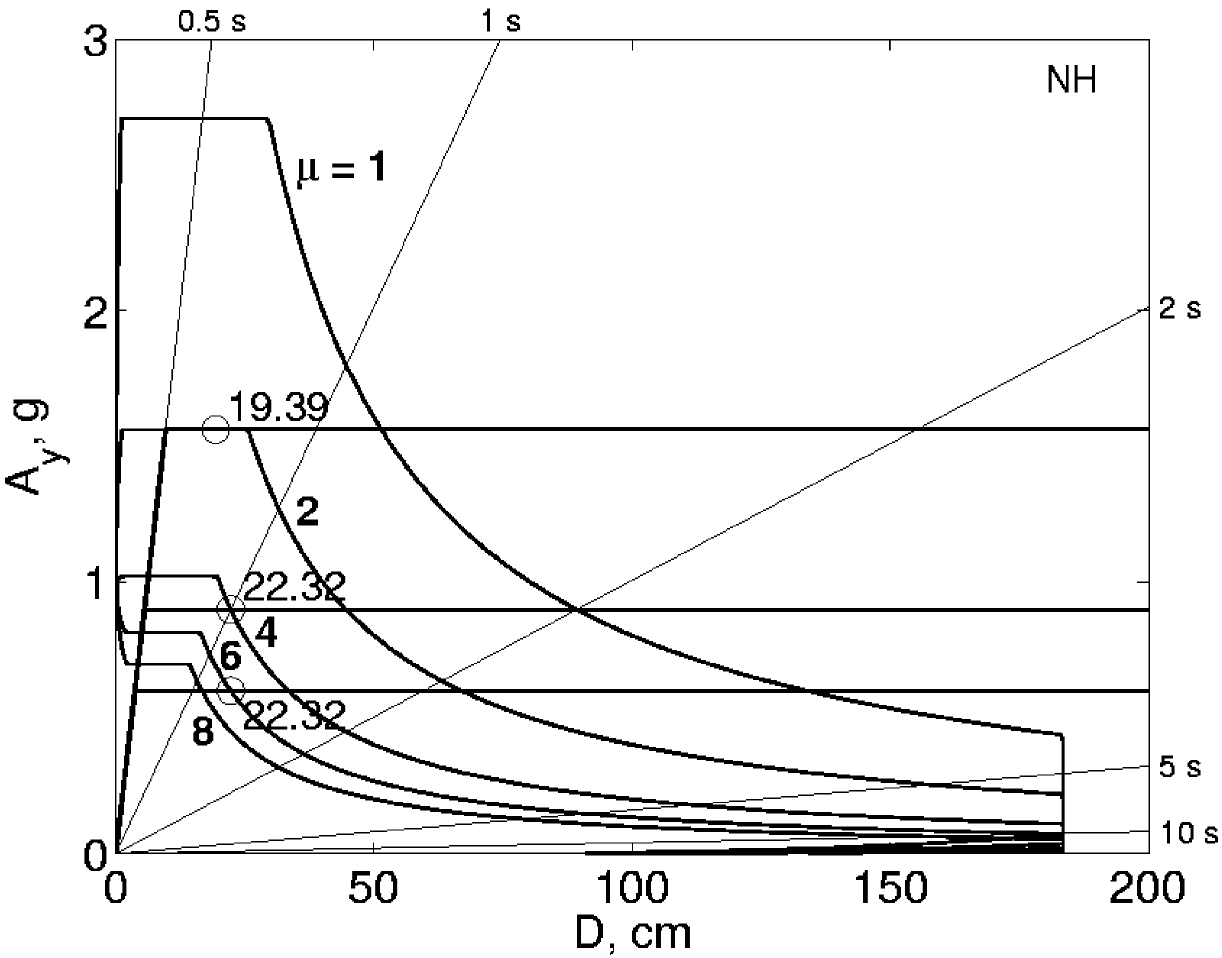
- Plot capacity and demand diagrams in *A-D* format
- Yielding branch of capacity diagram intersects the demand diagram for several μ
- **At relevant intersection point, μ from the two diagrams should match**
- Interpolate between two μ values or plot demand diagrams at finer μ values if necessary



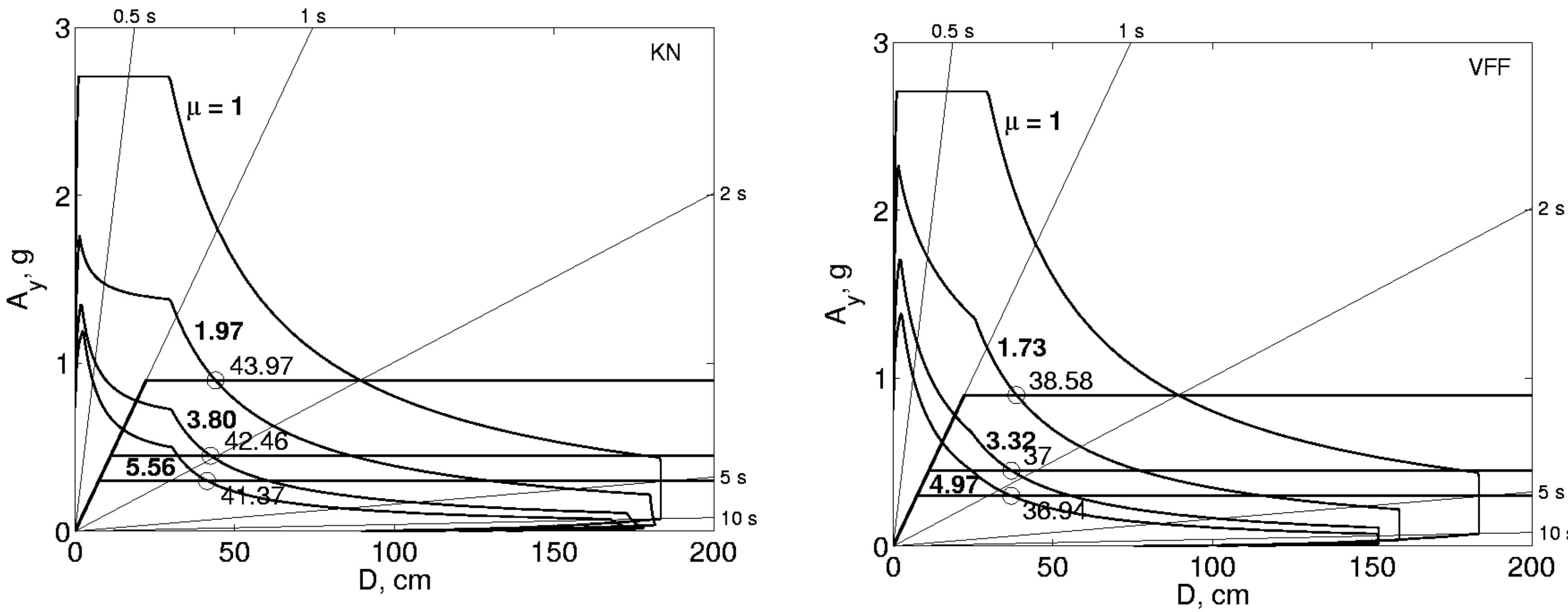
Improved Procedure-A Analysis of System 2



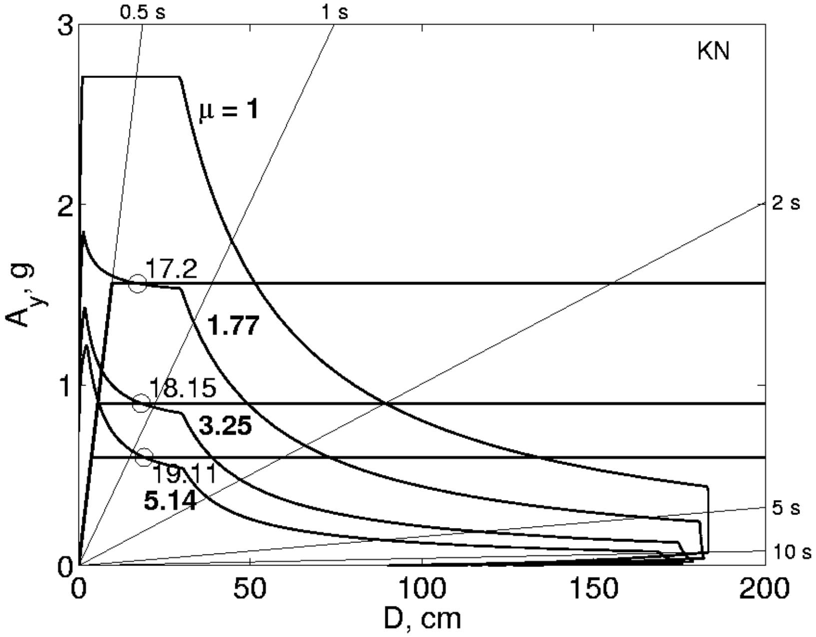
Improved Procedure-A Analysis of Systems 1 to 3

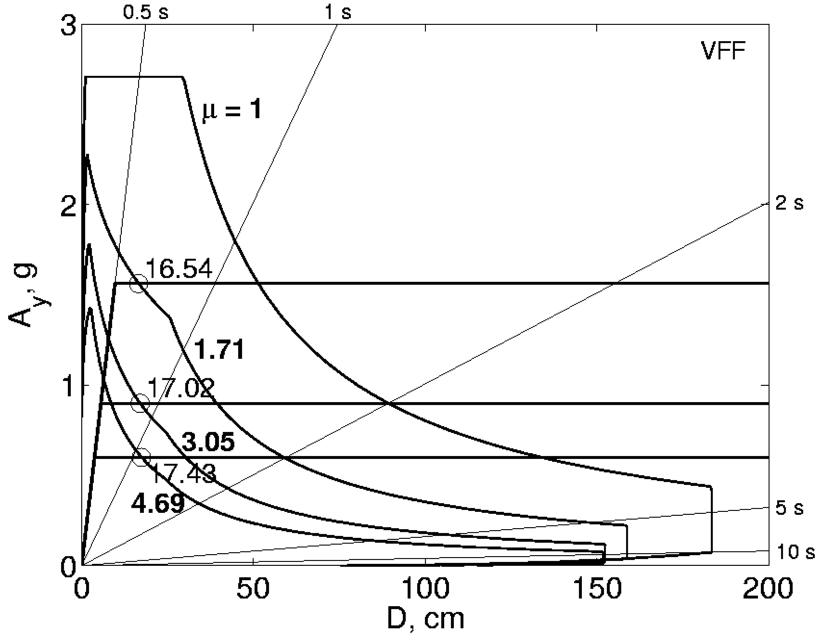


Improved Procedure-A Applied to Other Inelastic Design Spectra



Improved Procedure-A Applied to Other Inelastic Design Spectra



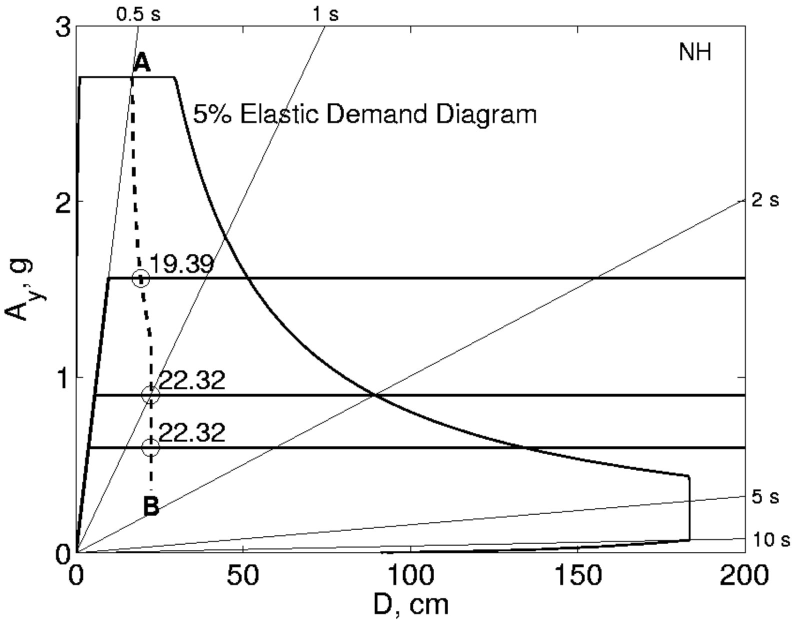


Improved Procedure-A

- Gives deformations consistent with the selected design spectrum
- Retains graphical feature of ATC-40 Procedure-A
  - ➔ Desired deformation is at intersection of capacity and demand diagrams
- Demand diagram used is different
  - ➔ Constant-μ demand diagram in improved procedure
  - ➔ Elastic demand diagram in ATC-40

Improved Procedure-B

- Plot capacity diagram
- Plot $D - A_y$ pairs to generate curve A-B
  - ➔ Assume expected μ
  - ➔ Determine A_y from inelastic design spectrum
  - ➔ Calculate D
- Find intersection of A-B and the capacity diagram



Improved Procedure-B

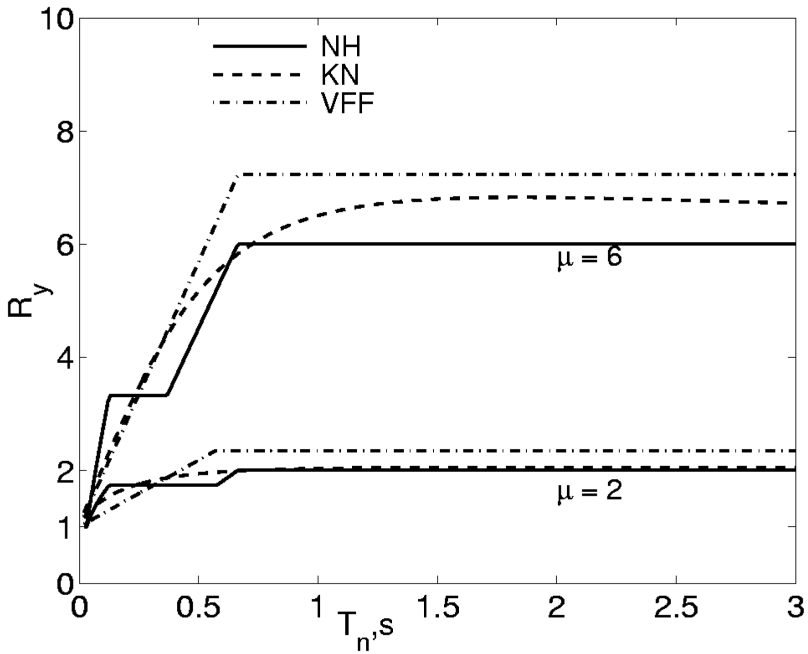
- Gives deformations consistent with the selected design spectrum
- Retains graphical feature of ATC-40 Procedure-B
  - ➔ Desired deformation is at intersection of curve A-B and the capacity diagram
- Different systems are analyzed to obtain a point on curve A-B
  - ➔ Inelastic system in improved procedure
  - ➔ Equivalent elastic system in ATC-40

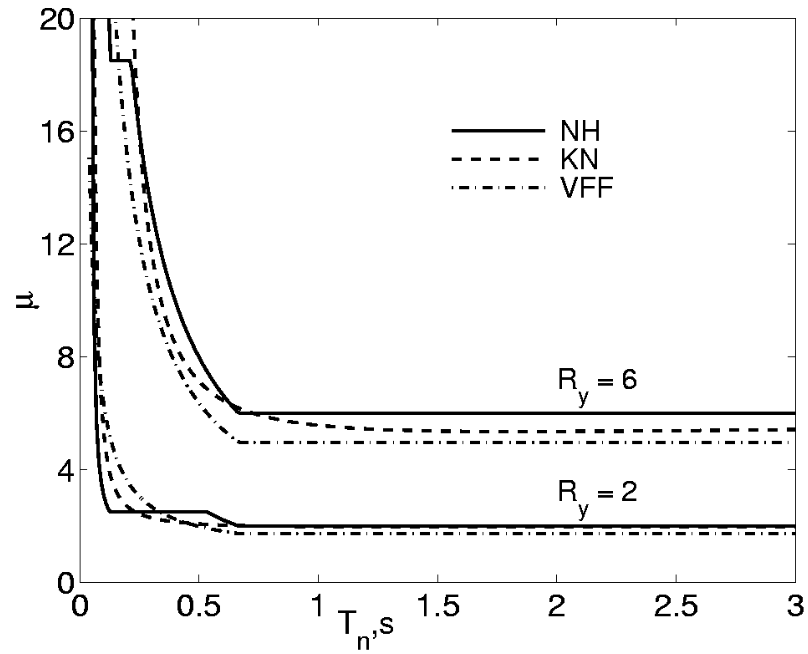
Numerical Version of Improved Procedure

- Compute $R_y = f_0 \div f_y = A \div A_y$
- Determine μ from available R_y–μ–T_n relations
  - ➔ Newmark-Hall
  - ➔ Krawinkler-Nassar
  - ➔ Vidic-Fajfar-Fischinger
  - ➔ Others
- Estimate deformation demand: $D = \mu D_y$

Three Different R_y–μ–T_n Relations

- R_y versus T_n for selected μ
- μ versus T_n for selected R_y





Numerical Version of Improved Procedure Analysis of System 3

- Given: T_n = 0.5s, ζ = 5%, A_y = 1.56 g
  - ➔ A = 2.71 g for system to remain elastic
  - ➔ D_y = 9.7 cm
- Newmark-Hall Inelastic Design Spectrum
- Find: D

1. Ry = 2.71 ÷ 1.56 = 1.73.
2. $\mu = (1+1.73^2) \div 2 = 2.0$

   Obtained from Newmark-Hall R_y–μ–T_n relations
3. $D = 2.0 \times 9.7 = 19.4$ cm

Conclusions

- ATC-40 method is inaccurate
  - ➔ Underestimates deformation significantly over a wide range of T_n and μ values
  - ➔ $D \cong$ Half of "Exact"
- ATC-40 method is deficient compared to elastic spectrum in velocity and displacement regions

Conclusions

- Improved capacity-demand-diagram methods, based on well-known constant ductility design spectrum, have been developed
  - ➔ Improved Procedure-A
  - ➔ Improved Procedure-B
  - ➔ Numerical Version of Improved Procedure
- Improved procedures give deformation consistent with the selected design spectrum

Conclusions

- Improved Procedure-A retains graphical feature of ATC-40 Procedure-A
  - ➔ Desired deformation is at intersection of capacity and demand diagrams
- Demand diagram used is different
  - ➔ Constant-μ demand diagram in improved procedure
  - ➔ Elastic demand diagram in ATC-40

Conclusions

- Improved Procedure-B retains graphical feature of ATC-40 Procedure-B
  - Desired deformation is at intersection of curve A-B and the capacity diagram
- Different systems are analyzed to obtain a point on curve A-B
  - Inelastic system in improved procedure
  - Equivalent elastic system in ATC-40

Conclusions

- Numerical Version of Improved Procedure is convenient if graphical feature is not needed
  - ➔ Based on R_y–μ–T_n relations
  - ➔ Gives essentially the same values of deformation as graphical implementation